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ABSTRACT

The books in this series provide an essential ingredient in the transition from concrete mathematical experience to the symbolic reasoning used in algebra. They aim to guide the student in building and extending manipulative patterns, collecting numerical data, and analyzing the results. The main objective of the activities in this book is for the student to be able to build any design of the pattern. The activities allow students to use cubic blocks to build and analyze the given designs, then determine the relationship between the number of the design and the number of cubes necessary to build the design. Exploration and discovery of individual solution strategies are encouraged. The book is laid out in an easy-to-use format with 50 pattern pages (divided into three sections), an appendix (including teaching guide and a glossary), and an answer key. Within the five sections included, patterns generally progress from the simple arrangements that build confidence to the more complex and creative patterns that inspire students to combine their imagination and their mathematical skills. (JRH)

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CUBES

Building Algebraic Thinking with Progressive Patterns

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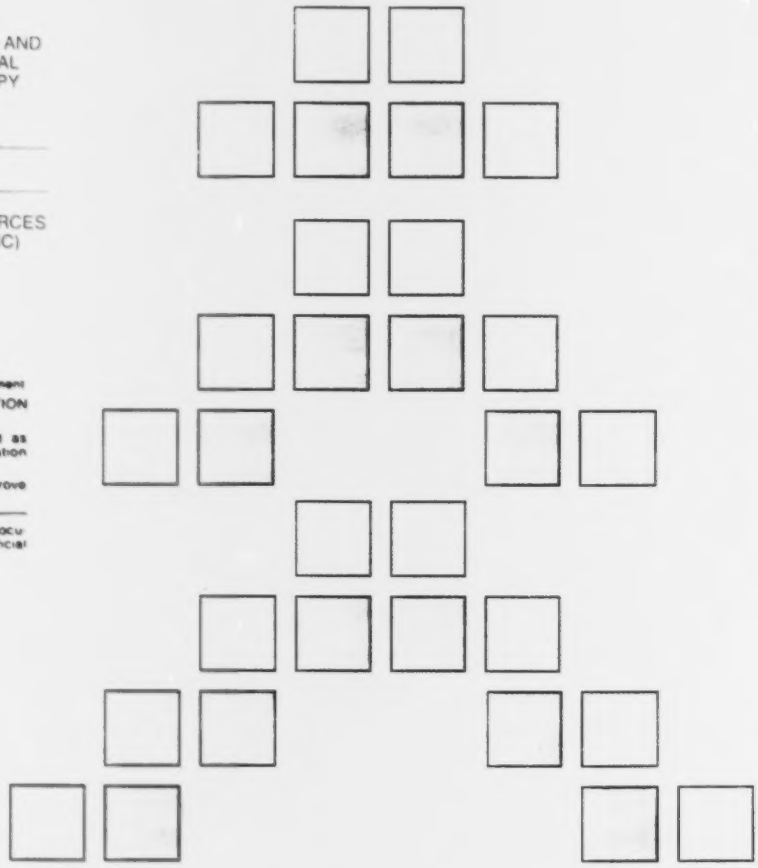

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CUBES

Building Algebraic Thinking with Progressive Patterns

Bob Willcutt

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ABOUT THE PROGRAM

Goals

The books *Building Algebraic Thinking with Progressive Patterns* (*CUBES*, *RODS*, and *BLOCKS*) provide an essential ingredient in the transition from concrete mathematical experience to the symbolic reasoning used in algebra. The program guides the student in building and extending manipulative patterns, collecting numerical data, and analyzing the results. The activities engage creativity and provide positive early experiences with a variable, helping students make a smooth transition into the study of algebra. *CUBES* focuses on the use of cubic blocks (such as Unifix cubes); *RODS*, on Cuisenaire-type rods; and *BLOCKS*, on pattern blocks (varied shapes). The books are self-contained and do not need to be used in any special order.

Building Algebraic Thinking is a discovery-based approach, using a questioning strategy to provide problem solving experiences as recommended by the National Council of Teachers of Mathematics (N.C.T.M.). By following the given strategy and building patterns with manipulative materials, students develop the skills needed to

- substitute for a variable in an equation
- write generalized equations using a variable
- find creative solutions based on geometric reasoning
- combine like variables and numbers
- factor
- reverse process

Throughout the book, exploration and discovery of individual solution strategies are encouraged. Ultimate success is attained when students can create and solve designs of their own.

Objective/Methods

On each activity page, the objective is for the student to be able to build any design of the pattern. To do this, the student uses blocks to build and analyze the given designs and then determines the relationship between the number of the design and the number of blocks necessary to build the design. As the pattern progresses from the first design (design number 1) to the 4th and 5th designs, students will notice that the number of blocks required to build each successive design increases in a predictable way. This results in a "generalization," a general equation, using the variable n, that applies to all designs of the pattern. The design number is substituted for n to get the number of blocks needed to build the design. For example, on page 1, the student finds that multiplying the design number by two gives the number of blocks needed to build that design of the pattern (design 3 requires $3 \times 2 = 6$ blocks). Substituting n for the design number, the student can give a generalization of $n \times 2 =$ number of blocks needed. To find the number of blocks needed for building design 12, substitute 12 for n to get $12 \times 2 = 24$ blocks.

The manipulative activities program can be used with individuals, small groups, and whole classes. Questions on the activities, or pattern, pages are let-

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tered sequentially, but the suggested questions do not have to be answered in the given order. There is no "right" way for students to come to the correct generalization, and they should be encouraged to find their own methods.

Based on the columns on the Number of Blocks page, students may recognize a pattern of regularly increasing numbers. If the numbers always grow by 2, the student puts that number before n to get $2n$ for the total number of blocks. This "constant difference" method is a systematic and efficient tool for solving even the most challenging generalizations.

On the other hand, "visual" learners may best see the generalization by relating the physical, or spatial, arrangements of blocks to the total number of blocks. If there are 2 blocks in each of 5 rows for design 5, there are $2n$ blocks in all. If the generalization is not readily seen in the given arrangement, students may want to rearrange blocks to make the generalization more obvious.

Some students find the generalization by the constant difference method and then explain how it is verified by the block arrangement. Enlisting the creative imagination (What does this design remind you of?) can enhance the learning process. Whatever methods are used, it is important that students share ideas and see that there can be a variety of creative ways to find generalizations.

Using the Book

Building Algebraic Thinking is laid out in an easy-to-use format with 50 pattern pages (divided into 3 sections), an appendix (including 3 reproducible Number of Blocks pages, a teaching guide, and a glossary), and an answer key. Each pattern includes a progression (numbered 1 through n) of designs, each consistent with the overall pattern. All pattern problems involve one variable (n) and require one, two, or three types of manipulative objects (shown as squares representing cubic blocks). The problems become increasingly more complex in each of the book's three sections. Answers are found in the answer key at the end of the book.

Each student (or cooperative group) should have one Number of Blocks page for each pattern page and enough blocks to build as many designs as necessary; the number of designs built may vary with the students. (See the appendix for reproducible Number of Blocks pages.) Following the illustrated pattern as a model, students use the blocks to build the designs; the designs can be used to generate the number data for the Number of Blocks page. For each design built, students must record the numbers of blocks needed. This task reinforces the importance of collecting correct data, as do many science-based activities, because one wrong entry will break the emerging pattern!

The questions on each pattern page help direct students in analyzing relationships between the numbers of the designs and the numbers of blocks to determine and express a generalized equation using the variable n. To foster students' experience with a variable, you might ask, "What would happen if I decided to use n to stand for any number? The n could be 10 or 580 or whatever we want it to be. What will we put in the blocks column if we use n for the design number?" (See chart at right.) The answer will be $2n$, or $n + n$.

Design	□s
1	2
2	4
3	6
4	8
5	10
⋮	⋮
10	20
⋮	⋮
20	40
⋮	⋮
n	n + n or 2n

Students apply the generalization to determine the number of blocks required for any design of the pattern. Both forward processing (design 2 takes how many blocks?) and reverse processing (which design will take 20 blocks?) are required.

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Even if the students do not yet have the tools for formal algebraic operations, by exploring with concrete materials, students form a foundation for understanding later algebraic operations.

Within the three sections of the book, patterns generally progress from the simple arrangements that build confidence to the more complex and creative patterns that inspire students to combine their imagination and their mathematical skills. Some of the final exercises (exploring quadratic solutions) may seem very difficult, but students may use calculators to aid exploration (trial and error is encouraged). Though some students will no longer actually need to build the patterns after a few pages, others may continue to need the concrete experience into the final section.

Because *Building Algebraic Thinking* is a discovery-based program, it is recommended that teachers use a flexible approach, gearing the program to students' abilities. For those who feel the need for specific guidance, however, a questioning guide is provided in the appendix.

Specific information about each of the three pattern sections can be found in the chart that follows.

CONTENTS OF MANIPULATIVE ACTIVITIES PAGES

	Number/Types Manipulatives	Design Formats	Solution Types	Skills/Features	Suggestions
SECTION 1 (Pages 1–20)	1 Type: white cubic blocks	From very simple, regular, and rectangular designs to simple, irregular, and more creative designs; primarily symmetrical	Pages 1–19: Linear Page 20: Quadratic	Pages 1–19: Basic multiplication Pages 1–6: Reverse processing; Building confidence (mental arithmetic, class discussion, exploration) Pages 3–5: Rearranging blocks (promotes creativity and connnects geomety and algebra)	• For those who need concrete experiences, actually building the designs is crucial • Encourage exploration of the relationship between physical arrangement and generalization • Let students discover the relationship between the constant difference and the coefficient of "n" (do not directly show them) • Extensions: have students (A) create own patterns (B) graph data from pages 1–6 on an "xy" graph (see Extension Activities)
SECTION 2 (Pages 21–40)	2 Types: white and black cubic blocks	From simple, regular, and rectangular designs to regular and somewhat complex designs; primarily symmetrical	Pages 21–37: Linear Pages 38–40: Quadratic	Combining like terms Factoring Reverse processing Constant difference method (becomes more efficient) Seeing a connection between the constant difference and the coefficient is not crucial at this stage	• Continue to encourage analysis of physical arrangement • Give the first 3 designs of a creative pattern and have students build next one (some may come up with a divergent, though still consistent, design). Discuss the question, When is a pattern clearly established?
SECTION 3 (Pages 41–50)	3 Types: white, black, and gray cubic blocks	From regular designs to artistically creative designs (simple rectangles for advanced concepts); primarily symmetrical	Pages 41–47: Linear Pages 48–50: Quadratic (very challenging)	Combining like terms Reverse processing Combining creativity and mathematics skills (promoted by artistically arranged pattern designs) Many students may not need the actual building experience; some may need an intermediate step to the abstract (drawing symbols such as X's or O's)	• For a good spatial activity (similar to those on many standardized tests), show the first three designs of a pattern and have students create the next one or more designs • Continue to encourage analysis of physical arrangement and also constant difference (the latter becomes more efficient as block arrangement becomes obscure) • Have students create their own patterns (see Extension Activities)

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Manipulatives

It is very important that students see how the manipulative designs generate the associated numerical pattern; therefore, they must actually build the designs before going on to the number work. Not all students will continue the manipulative work throughout the book, but for those who need concrete experiences (and even adults do!), building and extending the designs is crucial and will provide the primary benefit. Students with special needs get a lot of satisfaction from continually extending the patterns. Students can always return to the numerical work at a later date.

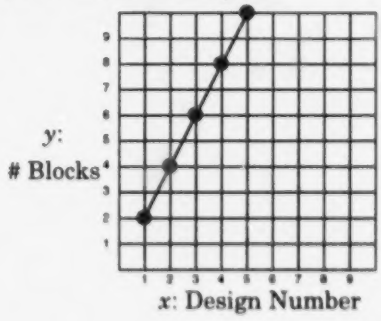
Each student should have access to enough blocks to build the designs. Commercial materials are available from various manufacturers, but a sufficient supply can probably be found close at hand. Manipulatives can be of many different forms: color tiles, color cubes, unit cubes, or any similar product will work well. Homemade cardboard squares, paper clips, beans, recycled materials, or chips are other possibilities. Different colors are helpful for building patterns that require more than one type of manipulative; the main thing is to have enough manipulatives for students to do the actual building.

Extension Activities

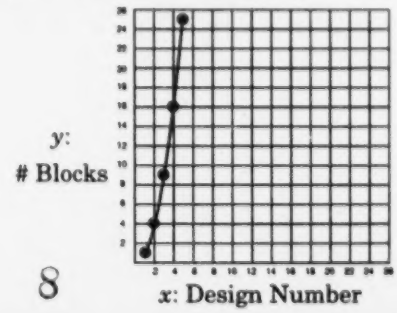
The activities below can provide a valuable extension to the regular manipulative activities.

- Creating patterns: Have students make their own designs adhering to the criteria for a pattern. Remind them that progressive designs must be consistent with the overall pattern; patterns must be developed to the point where, based on the given designs, there is only one subsequent design that can logically follow. Have students share their own patterns with other students or their families and challenge them to solve for the generalization. A copy of the pattern will make an appropriate entry in a student's learning portfolio. Also, each student could contribute one pattern page to a class pattern book.
- Graphing the equations: Give students graph paper and instruct them to label and mark the x-axis with design numbers and the y-axis with the number of blocks used. Have them plot the number of blocks for each of a specified number of designs and join the points. Guide them in recognizing the relationship of the line (straight or curved) to the type of solution required for the generalized equation (linear or quadratic). See graphing examples below.

Linear Function Graph



Quadratic Function Graph



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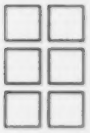
PATTERN 1

1

2

3



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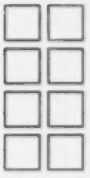

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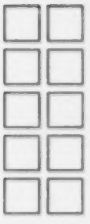

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10

⋮

20

⋮

n

A. How many □s will we need for the 16th design? _____

B. As you build the pattern and fill in the blanks on the Number of Blocks page, notice how the numbers change from one design to the next. Describe the number pattern.

_____

_____

C. Will we ever need exactly 79 □s to build a design? _____ Why?

_____

_____

D. We will need _____ □s for the 67th design.

E. With 84 □s, we can build the _____ design.

F. The outside dimensions of the rectangles created in designs 1–5 are listed below. Fill in the missing dimensions.

Design	Base × Height
1	2×1
2	2×2
3	2×3
4	2×4
5	2×5
8	_____
10	_____
20	_____
n	_____

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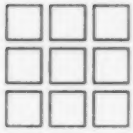
PATTERN 2

1

2

3



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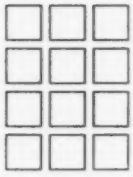

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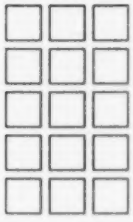

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10

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20

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n

A. How many □s will we need for the 17th design? _____

B. As you build the pattern designs and fill in the blanks on the Number of Blocks page, notice how the numbers change from one design to the next. Describe the number pattern.

_____

_____

C. Will we ever use exactly 91 □s to build a design? _____ Why?

_____

_____

D. When we use a total of 27 □s, we can build the _____ design.

E. The 59th design will use exactly _____ □s.

F. Below are the outside dimensions of the rectangles created in designs 1–5. Fill in the missing dimensions.

Design	Base × Height
1	3 × 1
2	3 × 2
3	3 × 3
4	3 × 4
5	3 × 5
9	_____
10	_____
20	_____
n	_____

10

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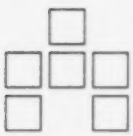
PATTERN 3

1

2

3



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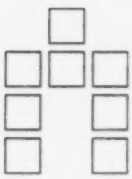

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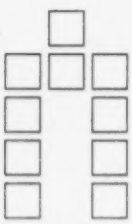

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n

A. How many □s will we need for the 15th design? _____

B. As you build the pattern, how do the numbers of blocks change from one design to the next (see the Number of Blocks page)? Describe the resulting number pattern.

_____

_____

C. With 50 □s, we can build the _____ design.

D. Will we ever use exactly 65 □s to build a design? _____ Why?

_____

_____

E. We will need _____ □s for the 89th design.

F. With 378 □s, we can build the _____ design.

G. How was the pattern on page 1 similar to the pattern on this page?

_____

_____

How was it different?

_____

_____

Explain one way you could rearrange any design on this page so it is just like a design on page 1.

_____

_____

_____

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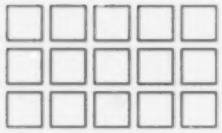
PATTERN 4

1

2

3



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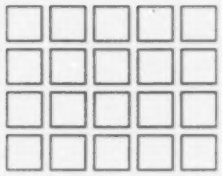

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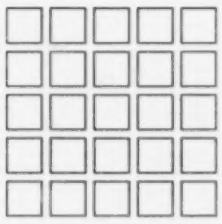

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10

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20

⋮

n

A. How many □s will we need for the 19th design? ____

B. As you build the pattern, how do the numbers of blocks change from one design to the next (see the Number of Blocks page)? Describe the resulting number pattern.

_____

_____

C. How many blocks do we add to each design as we build successive designs? ____ How do we use that number in our generalization for *n* blocks?

_____

_____

D. A total of 190 □s will be used for the ____ design.

E. Will we ever use exactly 224 □s to build a design? ____ Why?

_____

_____

F. Below are the outside dimensions of the rectangles created in designs 1–5. Fill in the missing dimensions.

Design	Base × Height
1	5 × 1
2	5 × 2
3	5 × 3
4	5 × 4
5	5 × 5
8	____
10	____
20	____
n	____

12

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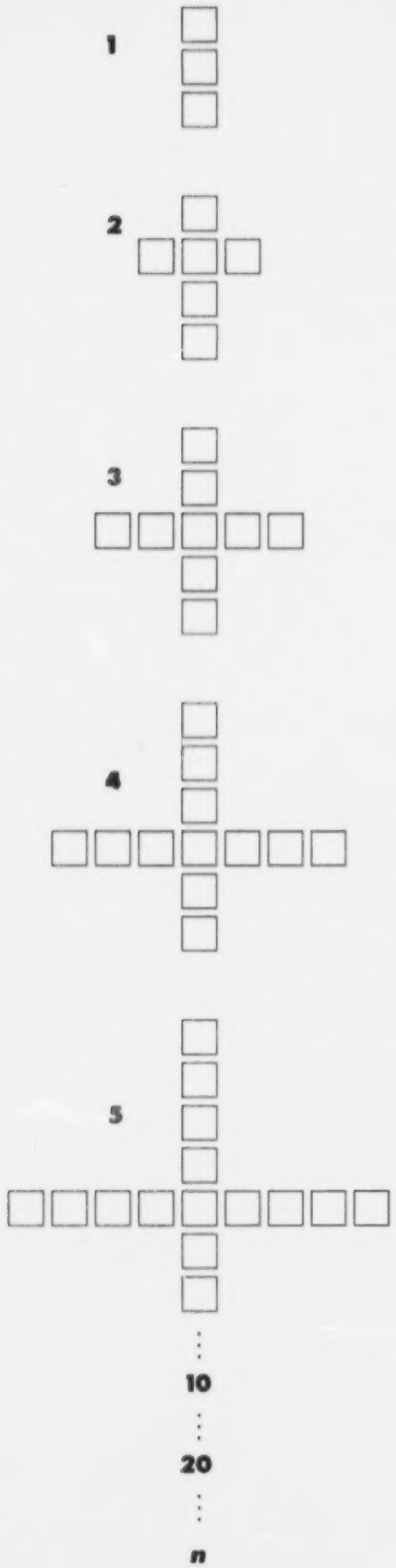
PATTERN 5



A. For the 13th design, how many □s will we need? _____

B. As you build the pattern, how do the numbers of blocks change from one design to the next (see the Number of Blocks page)? Describe the resulting number pattern.

_____

_____

C. How many blocks did you add to each design? _____ How is that constant number used in the generalization for n □s?

_____

_____

D. With 90 □s, we can build the _____ design.

E. Will we ever use exactly 35 □s to build a design? _____ Why?

_____

_____

F. With 303 □s, we can build the _____ design.

G. This page uses the same number of □s and has the same generalization as the designs on page 2. Explain one way you could rearrange the □s in any design on this page to create a design on page 2.

_____

_____

_____

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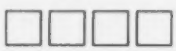
PATTERN 6

1



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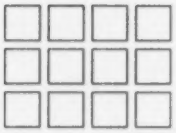

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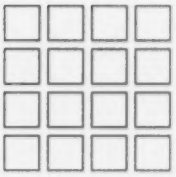

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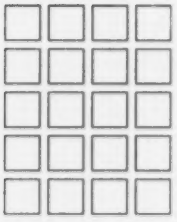

⋮

10

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⋮

n

A. The 18th design will use how many □s?
____

B. As you build the pattern, how do the numbers of blocks change from one design to the next (see the Number of Blocks page)? Describe the resulting number pattern.

_____

_____

C. Will we ever use an odd number of □s to build a design? ____ Why?

_____

_____

D. A total of 128 □s, will be used for the ____ design.

E. The 59th design will use ____ □s.

F. The outside dimensions of the rectangles created in designs 1–5 are shown below. Fill in the missing dimensions.

Design	Base × Height
1	4×1
2	4×2
3	4×3
4	4×4
5	4×5
8	____
10	____
20	____
n	____

14

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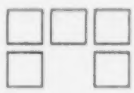
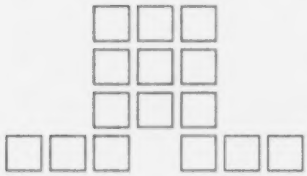
PATTERN 7

1

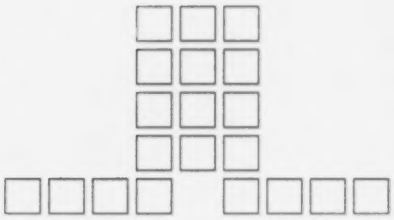


2

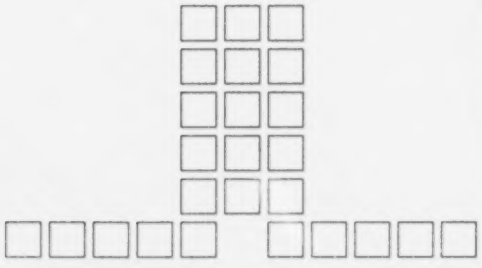
3



4



5



⋮

10

⋮

20

⋮

n

A. How many □s will we use for the 15th design? _____

B. Explain one way you can find the generalization for *n* by studying the block arrangements.

_____

_____

C. Will any design use exactly 42 □s? _____ Why?

_____

_____

D. How many □s do we add to each successive design? _____ How do we use this number in our generalization for *n* □s?

_____

_____

E. When we use exactly 95 □s, we can build the _____ design.

F. The 80th design will use _____ □s.

G. For building the _____ design, 585 □s will be just right.

H. This page uses the same number of □s and has the same generalization as the designs used on page 4. Compare the pictures on this page with those on page 4 and describe how the designs are basically the same.

_____

_____

_____

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PATTERN 8

1

2

3

4

5

⋮

10

⋮

20

⋮

n

A. How many □s will we need for the 18th design? ____

B. As you build the pattern, how do the numbers of blocks change from one design to the next (see the Number of Blocks page)? Describe the resulting number pattern.

_____

_____

C. Will we ever use exactly 56 □s to build a design? ____ Why?

_____

_____

D. With 49 □s, we can build the ____ design.

E. Will the design ever create a rectangle? ____ Why?

_____

_____

F. We will need ____ □s for the 65th design.

G. With 531 □s, we can build the ____ design.

H. Explain one way that you can find the generalization for *n* by looking at the arrangement of blocks.

_____

_____

_____

16

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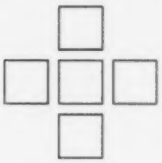
PATTERN 9

1

2



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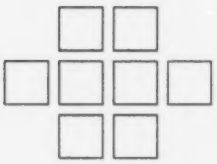
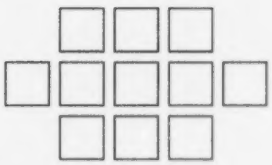
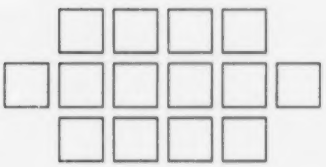


4



5



⋮

10

⋮

20

⋮

n

A. For the 15th design, how many □s will we need ? _____

B. How many blocks do we add to each design as we build successive designs? _____ How do we use that number in our generalization for *n* blocks?

_____

_____

C. Will the 40th design use an even number of blocks or an odd number of blocks? _____ Why?

_____

_____

D. A total of 149 □s will be used for the _____ design.

E. Below are the dimensions of the inside rectangles created in designs 1–5. Fill in the missing dimensions.

Design	Base × Height
1	0 × 3
2	1 × 3
3	2 × 3
4	3 × 3
5	4 × 3
8	_____
10	_____
20	_____
n	_____

How do the dimensions of the inside rectangle (plus the two outside blocks) show the generalization for *n*?

_____

_____

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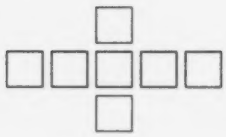
PATTERN 10

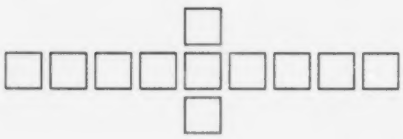
1

2

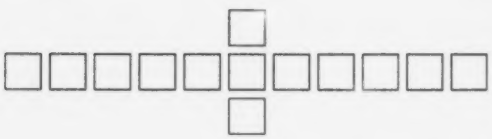


3

4



5



⋮

10

⋮

20

⋮

n

A. We will need _____ □s for the 22nd design.

B. How many blocks do we add to build successive designs? _____ How do we use that number in our generalization for *n* blocks?

_____

_____

C. Explain one way you can find the generalization for *n* by looking at the arrangement of blocks.

_____

_____

D. A total of 203 □s will be used for the _____ design.

E. The 65th design will use _____ □s.

F. To create the pattern on page 8, we used successive odd numbers of blocks. What do you notice when you compare the numbers from Pattern 8 with the numbers from Pattern 10 (see chart below)?

_____

_____

Design #	Page 8	Page 10
1	1	5
2	3	7
3	5	9
4	7	11
5	9	13
10	19	23
20	39	43
n	2n – 1	

How do the generalizations compare? (Fill in the generalization for page 10.)

_____

_____

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PATTERN 11

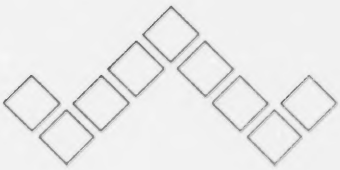
1

2

3

4

5



⋮

10

⋮

20

⋮

n

A. How many □s will we use for the 13th design? _____

B. As you build the pattern, how do the numbers of blocks change from one design to the next (see the Number of Blocks page)? Describe the resulting number pattern.

_____

C. With exactly 29 □s, we can build the _____ design.

D. We will need _____ □s for the 50th design.

E. With 199 □s, we can build the _____ design.

F. For Pattern 1, we used successive even numbers of blocks. How does that number pattern compare with the pattern on this page (see chart below)?

_____

_____

Design #	Page 1	Page 11
1	2	1
2	4	3
3	6	5
4	8	7
5	10	9
10	20	19
20	40	39
n	2n	

How do the generalizations compare? (Fill in the generalization for page 11.)

_____

G. Describe how the generalization for *n* is shown in the arrangement of blocks.

_____

_____

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PATTERN 12

1

2

3

4

5

⋮

10

⋮

20

⋮

n

A. How many □s will we need for the 17th design? _____

B. As you build the pattern, how do the numbers of blocks change from one design to the next (see the Number of Blocks page)? Describe the resulting number pattern.

_____

_____

C. Will we ever use exactly 89 □s? _____ Why?

_____

_____

D. How many blocks do we add to each successive design? _____ How do we use that number in our generalization for n blocks?

_____

_____

E. A total of 358 □s will be used for the _____ design.

F. Below are the outside dimensions of the rectangles created in designs 1–5. Fill in the missing dimensions.

Design	Base × Height
1	1 × 2
2	3 × 2
3	5 × 2
4	7 × 2
5	9 × 2
9	____
10	____
20	____
n	____

How do the dimensions for the nth design compare to the generalization?

_____

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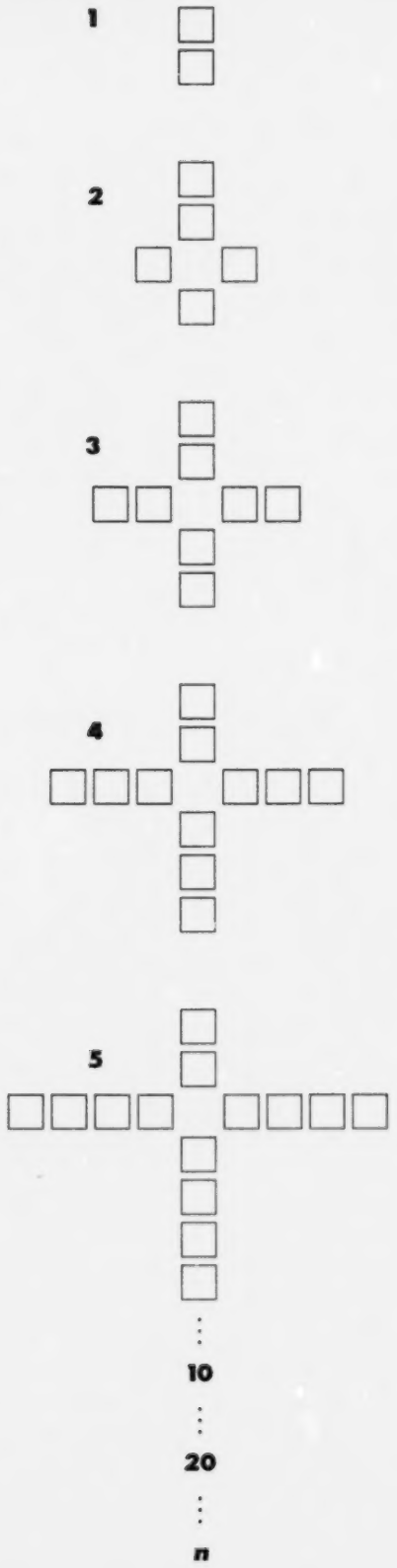
PATTERN 13



A. The 14th design will need _____ □s.

B. How many □s do we add to each successive design? _____ How do we use this number in our generalization for *n* blocks?

_____

C. With exactly 74 □s, we can build the _____ design.

D. Explain one way you can find the generalization for *n* by looking at the arrangement of blocks.

_____

_____

E. The 70th design will need _____ □s.

F. With 119 □s we can build the _____ design.

G. For Pattern 2, we used successive multiples of three. How do the number patterns on page 2 compare to the number patterns on this page (see chart below)?

_____

_____

Design #	Page 2	Page 13
1	3	2
2	6	5
3	9	8
4	12	11
5	15	14
10	30	29
20	60	59
n	3n	

How do the generalizations compare?

_____

_____

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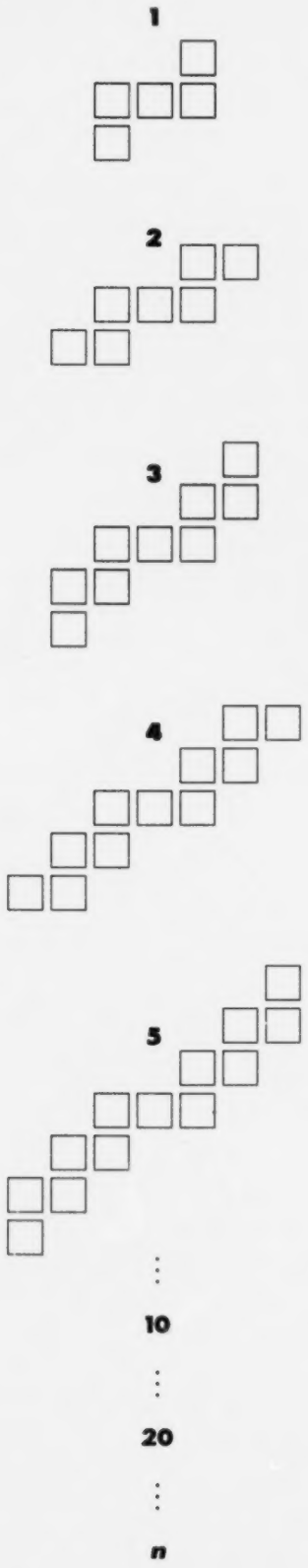
PATTERN 14



A. We will need _____ □s to build the 16th design.

B. How do the numbers of blocks change from one design to the next on the Number of Blocks page? Describe the pattern.

_____

_____

C. Explain one way you can find the generalization for *n* by looking at the arrangement of blocks.

_____

_____

D. The 50th design will need _____ □s.

E. With exactly 95 □s, we can build the _____ design.

F. For building the _____ design, 203 □s will be just right.

G. Like the designs on this page, the designs on page 8 use consecutive odd numbers (see the chart below). How do the two number patterns compare?

_____

_____

Design #	Page 8	Page 14
1	1	5
2	3	7
3	5	8
4	7	11
5	9	13
10	19	23
20	39	43
n	2n – 1	

How do the generalizations compare?

_____

_____

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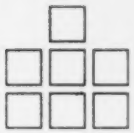
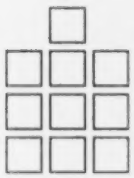
PATTERN 15

1

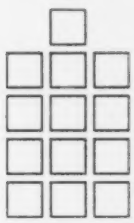
2



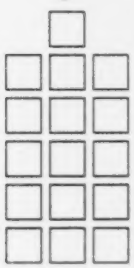
3



4



5



⋮

10

⋮

20

⋮

n

A. For the 23rd design, how many □s will we use? _____

B. How many □s do we add to each successive design? _____ How do we use that number in our generalization for *n* blocks?

_____

_____

C. Explain one way you can find the generalization for *n* by looking at the arrangement of blocks.

_____

_____

D. A total of 121 □s will be used for the _____ design.

E. The 83rd design will use _____ □s.

F. On page 5, we used successive multiples of three to solve the pattern. Use the chart below to explain how the number patterns on this page compare with those on page 5.

_____

_____

Design #	Page 5	Page 15
1	3	4
2	6	7
3	9	10
4	12	13
5	15	16
10	30	31
20	60	61
n	3n	

How do the generalizations compare?

_____

_____

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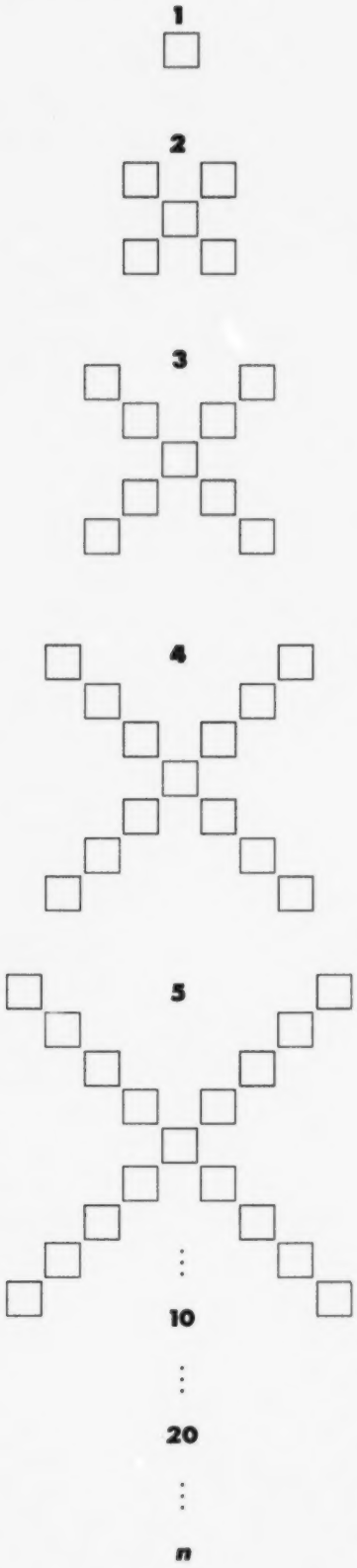
PATTERN 16



A. How many □s will we need for the 14th design? _____

B. Explain one way you can find the generalization for n by looking at the arrangement of blocks.

_____

_____

C. How many □s do we add to each successive design? _____ How do we use this number in our generalization for n □s?

_____

_____

D. With exactly 85 □s , we can build the _____ design.

E. If we use exactly 50 □s, can we build one of the designs? _____ . Why?

_____

F. The 80th design will use _____ □s.

G. Exactly 197 □s will fit into the _____ design.

H. On page 6, we used successive multiples of four. Compare the two patterns below, and describe how their generalizations compare.

_____

_____

Design #	Page 6	Page 16
1	4	1
2	8	5
3	12	9
4	16	13
5	20	17
10	40	37
20	80	77
n	4n	

24

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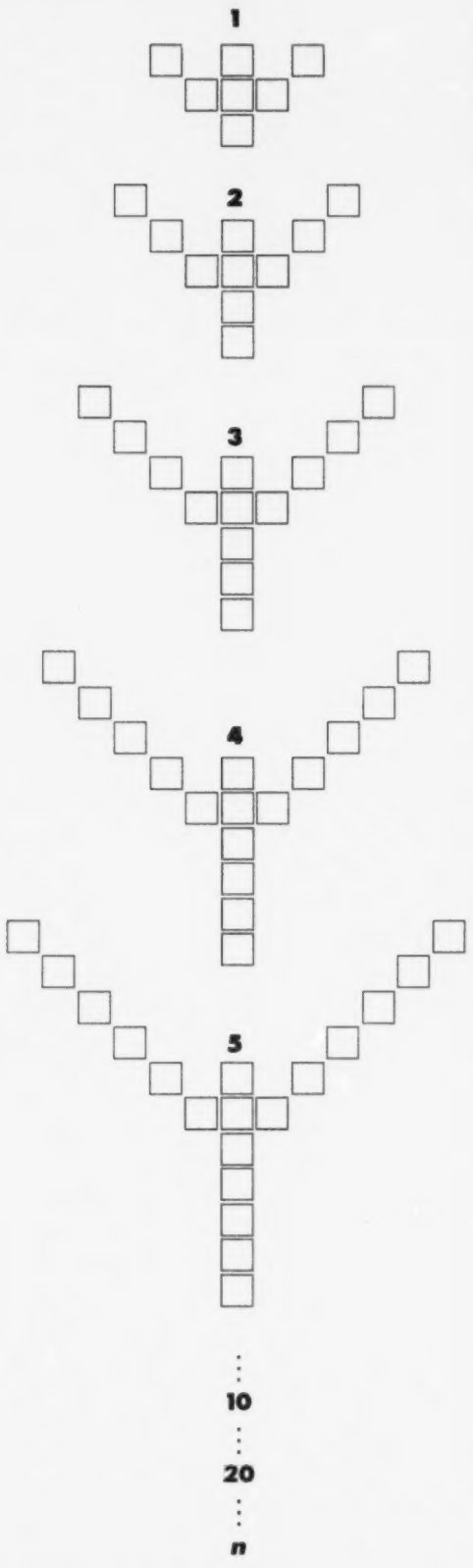
PATTERN 17



A. We will need how many □s for the 15th design? _____

B. How many □s do we add to each successive design? _____ How do we use this number in our generalization for *n* blocks?

_____

_____

C. With exactly 79 □s, we can build the _____ design.

D. Explain one way you can find the generalization for *n* by looking at the arrangement of blocks.

_____

_____

E. The 50th design will use _____ □s.

F. It will take 304 □s to fit into the _____ design.

G. Like the solutions on this page, the ones for page 9 increased by 3 □s for each design. Describe how the patterns compare (see chart below).

_____

_____

Design #	Page 9	Page 17
1	2	7
2	5	10
3	8	13
4	11	16
5	14	19
10	29	34
20	59	64
n	3n – 1	

How do the generalizations compare?

_____

_____

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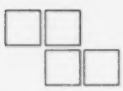
PATTERN 18

1

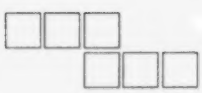
(No blocks)

2

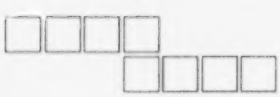
3



4



5



⋮

10

⋮

20

⋮

n

A. The 18th design will use how many □s?

____

B. How do the numbers of blocks change from one design to the next (see the Number of Blocks page)? Describe the resulting number pattern.

_____

_____

C. Look at the arrangement of blocks to explain one way you can find the generalization for n.

_____

_____

D. How many □s do we add to successive designs? _____ How do we use that number in our generalization for n blocks?

_____

E. A total of 198 □s will be used for the _____ design.

F. We solved the pattern on page 1 by using successive even numbers. How does the number pattern on page 1 compare to the pattern on this page (see chart below)?

_____

_____

Design #	Page 1	Page 18
1	2	0
2	4	2
3	6	4
4	8	6
5	10	8
10	20	18
20	40	38
n	2n	

How do the generalizations compare?

_____

_____

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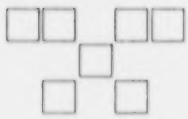
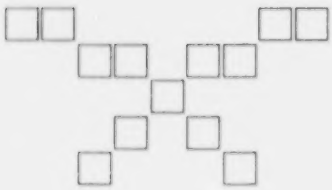
PATTERN 19

1

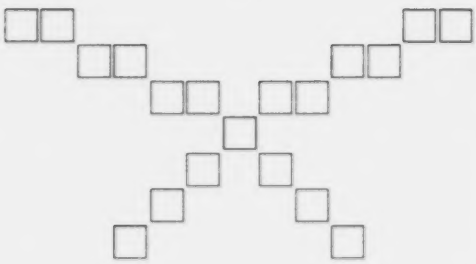
2



3



4



⋮

10

⋮

20

⋮

n

A. How many □s will we use for the 13th design? _____

B. How many □s do we add to each successive design? _____ How do we use this number in our generalization for *n* blocks?

_____

_____

C. Explain one way to find the generalization for *n* by looking at the arrangement of blocks.

_____

_____

D. The 50th design will use exactly _____ □s.

E. By using exactly 235 □s, we can create the _____ design.

F. Will we ever build a design using an even number of □s? _____ Why?

_____

_____

G. With 493 □s we can build the _____ design.

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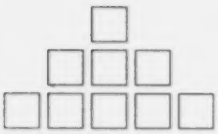
PATTERN 20

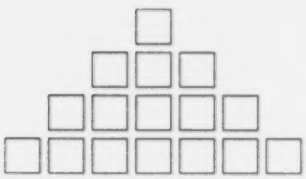
1

2

3



4



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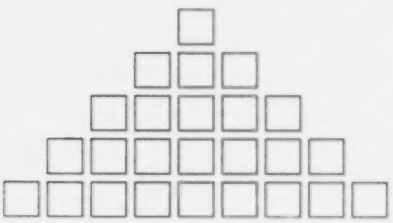

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A. For the 16th design, how many □s will we use? ____

B. We do not add the same number of □s to each design. What is the pattern we follow in adding □s to successive designs?

_____

_____

C. Look at the numbers of blocks used in successive designs on the Number of Blocks page. Describe the number pattern and tell what we call these numbers.

_____

_____

Why do we call them by this name?

_____

_____

D. With exactly 144 □s, we can build the ____ design.

E. Will we ever build a design using exactly 80 □s? ____ Why?

_____

_____

F. With exactly 625 □s we can build the ____ design.

G. Describe how you could rearrange each set of □s so that the design would create a square.

_____

_____

How does this square give us the generalization for *n*?

_____

28

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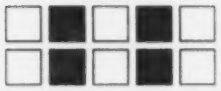
PATTERN 21

1

2



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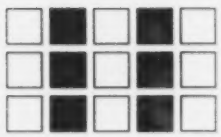

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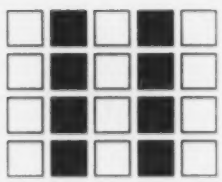

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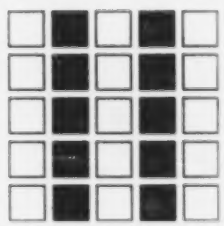

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A. The 25th design will use _____ ■s and _____ □s for a total of _____ blocks.

B. How does the physical arrangement of the ■s relate to the ■ generalization?

_____

How does the arrangement of the □s relate to the □ generalization?

_____

C. When we use 60 ■s, the total number of blocks used will be _____.

D. By how much do the numbers in the ■s column grow?

_____

in the □s column?

_____

in the Total column?

_____

How do the growing numbers relate to the generalization for *n* in each column?

_____

_____

E. Will our total number of blocks ever equal 110? _____ Why?

_____

_____

F. When our total number of blocks is 225, _____ will be □s.

G. Explain how the generalization for the total number of blocks equals the sum of the generalizations for the ■s and □s.

_____

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PATTERN 22

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A. The 25th design will use _____ ■s and _____ □s for a total of _____ blocks.

B. How does the physical arrangement of the ■s relate to the ■ generalization?

_____

How does the arrangement of the □s relate to the □ generalization?

_____

C. When we use 42 ■s, our total number of blocks will be _____.

D. Will we ever use a total of 86 blocks? _____ Why?

_____

_____

E. When the total number of blocks is 105, we will have _____ ■s and _____ □s.

F. The 200th design will use a total of _____ blocks.

G. For each design, the sum of the ■s and □s gives us the total number of blocks. Explain how the generalization for the ■s plus the generalization for the □s gives us the generalization for the total number of blocks.

_____

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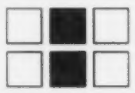
PATTERN 23

1



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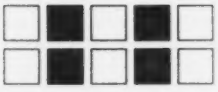

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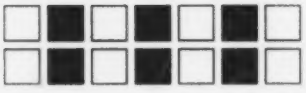

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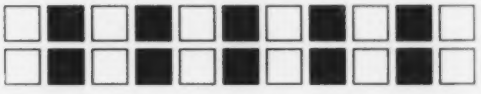

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n

A. The 18th design will use _____ ■s and _____ □s for a total of _____ blocks.

B. How does the physical arrangement of the ■s relate to the ■ generalization?

_____

How does the arrangement of the □s relate to the □ generalization?

_____

C. When we use 62 □s, the total number of blocks used will be _____.

D. Look at the number pattern for the total number of blocks. How many blocks do we add to each successive design?_____ How do we use that number in our generalization for the total number of blocks?

_____

_____

E. When we use a total of 162 blocks, _____ will be □s.

F. When we use 88 □s, the total number of blocks will be _____.

G. Explain how the final generalization for the total number of blocks equals the sum of the final generalizations for the ■s and □s.

_____

_____

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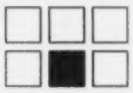
PATTERN 24

1



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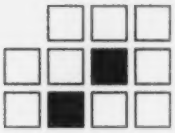

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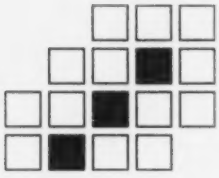

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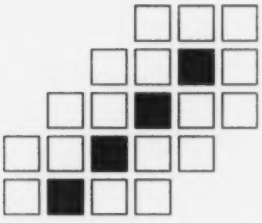

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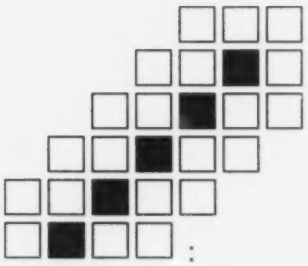

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n

A. The 28th design will use _____ ■s and _____ □s for a total of _____ blocks.

B. What pattern is created by the numbers of blocks used in successive designs for each of the following:

■s: _____

□s: _____

Total number of blocks:

_____

C. Will we ever use exactly 91 ■s? _____ Why?

_____

121 □s? _____ Why?

_____

D. By how much do the numbers grow in each column (Number of Blocks page)?

■s _____ □s _____ Total _____

How does the growing number relate to the generalization for each column?

_____

_____

E. When the total is 251, we will use _____ ■s and _____ □s.

F. When we use 189 □s, we will use _____ ■s.

G. Explain how adding the generalizations from each column gives the generalization in the total number of blocks column.

_____

_____

32

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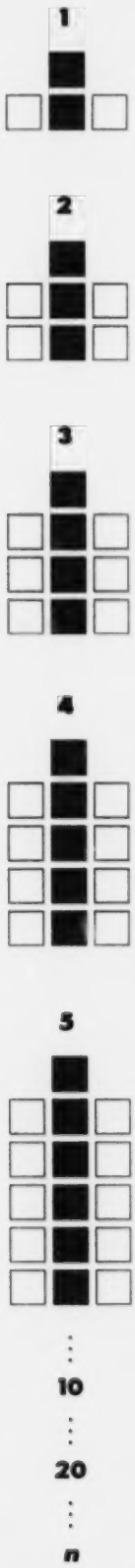
PATTERN 25



A. The 23rd design will use _____ ■s and _____ □s for a total of _____ blocks.

B. What pattern is created by the numbers of blocks used in successive designs for each of the following:

■s: _____

□s: _____

Total number of blocks: _____

_____

C. Describe how the physical arrangement of the □s shows the generalization for the □s.

_____

_____

D. In the Total column on the Number of Blocks page, by how much do the totals grow with each successive design?

_____

_____

How do we use that growing number in our generalization for the total number of blocks?

_____

_____

E. When the total number of blocks is 301, _____ will be □s.

F. Will the total number of blocks ever be 401? _____ Why?

_____

_____

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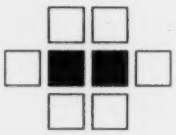
PATTERN 26

1



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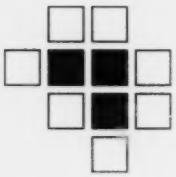

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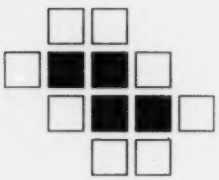

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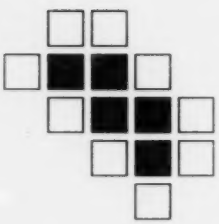

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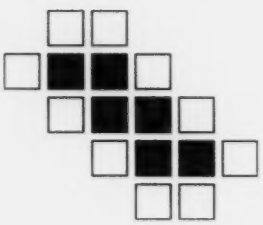

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n

A. The 34th design will use _____ ■s and _____ □s for a total of _____ blocks.

B. What patterns are formed by the numbers of blocks used for the ■s, the □s, and the total? What do we call these patterns?

_____

_____

C. When we use 70 ■s, we will have a total of _____.

D. Will we ever have a total of 189 blocks? _____ Why?

_____

_____

E. When we use 85 □s, we will use a total of _____ blocks.

F. When we use a total of 264 blocks, _____ will be ■s, and _____ will be □s.

G. The 350th design will use _____ ■s and _____ □s for a total of _____ blocks.

H. Explain how the generalization for the total number of blocks equals the sum of the generalizations for the ■s and □s.

_____

_____

_____

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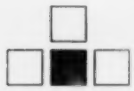
PATTERN 27

1



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n

A. The 23rd design will use _____ ■s and _____ □s for a total of _____ blocks.

B. How does the physical arrangement of the □s relate to the generalization for the □s?

_____

_____

C. By how much do the numbers grow in each column of the Number of Blocks page?

■s _____ □s _____ Total _____

How does the growing number relate to the generalization for *n* in each column?

_____

_____

D. When we use 43 ■s, the total number of blocks will be _____.

E. Will the total number of blocks ever equal 150? _____ Why?

_____

_____

F. When we use a total of 241 blocks, _____ will be ■s and _____ will be □s.

G. Explain how the generalization for the total number of blocks equals the sum of the generalizations for the ■s and □s.

_____

_____

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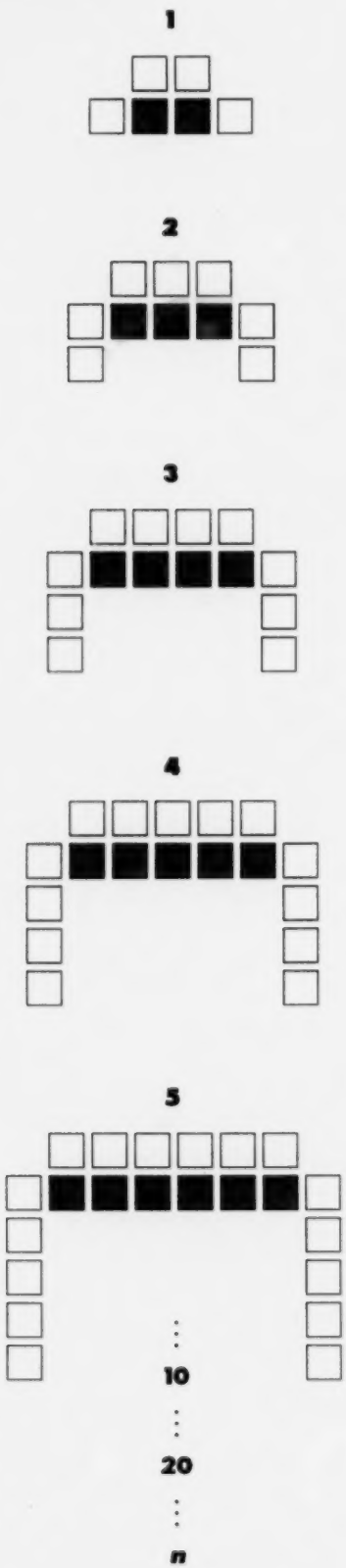
PATTERN 28



A. The 25th design will use _____ ■s and _____ □s for a total of _____ blocks.

B. Describe one way the physical arrangement of the □s can be used to explain the generalization for the number of □s.

_____

_____

C. When we use 39 ■s, the total number of blocks will be _____.

D. By how much do the numbers grow in each column of the Number of Blocks page?

■s _____ □s _____ Total _____

How does the growing number relate to the generalization for n in each column?

_____

_____

E. Will our total number of blocks ever equal 199? _____ Why?

_____

200? _____ Why?

_____

F. When we use a total of 262 blocks, _____ will be ■s.

G. Explain how the generalization for the total number of blocks equals the sum of the generalizations for the ■s and □s.

_____

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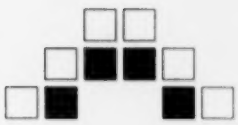
PATTERN 29

1

2



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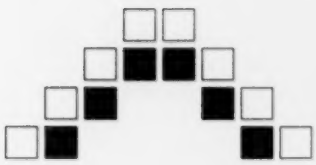

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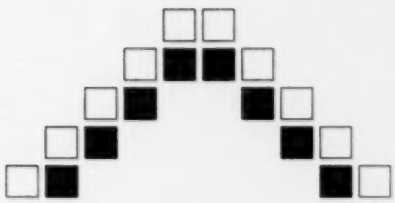

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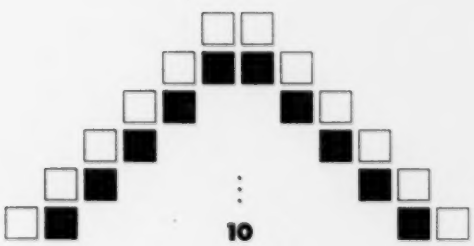

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A. When we use 46 ■s, we need _____ □s.

B. Explain one way you can rearrange the blocks to show the generalization for the total number of blocks.

_____

_____

C. When we use 200 ■s, the total number of blocks will be _____.

D. Will we ever have a total of 95 blocks? _____ Why?

_____

96 blocks? _____ Why?

_____

E. The total is 122 blocks. How many are □s? _____

F. By how much do the numbers grow in each column (Number of Blocks page)?

■s _____ □s _____ Total _____

How does the growing number relate to the generalization for *n* in each column?

_____

_____

G. The 87th design will use _____ ■s and _____ □s.

H. Explain how the generalization for the total number of blocks equals the sum of the generalizations for the ■s and □s.

_____

_____

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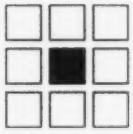
PATTERN 30

1



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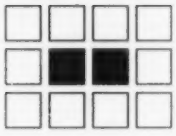

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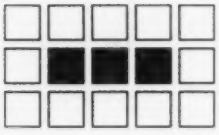

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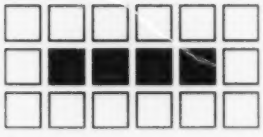

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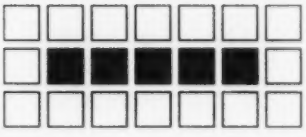

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n

A. The 30th design will use _____ ■s and _____ □s for a total of _____ blocks.

B. Describe one way the physical arrangement of the □s shows the generalization for the □s.

_____

_____

How can you find the generalization for the total number of blocks by looking at the block arrangement?

_____

_____

C. Will we ever use exactly 51 □s in a design? _____ Why?

_____

_____

D. When we use 46 ■s, we will use _____ □s.

E. Will we ever use a total of exactly 125 blocks to build a design? _____ Why?

_____

_____

F. When we use exactly 106 □s, the total number of blocks will be _____.

G. Below are the outside dimensions of the rectangles created in designs 1–5. Fill in the missing dimensions.

Design	Base × Height
3	5 × 3
4	6 × 3
5	7 × 3
9	_____
12	_____
20	_____
n	_____

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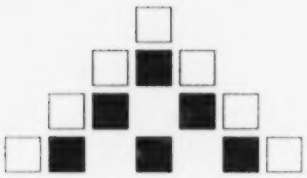
PATTERN 31

1

2



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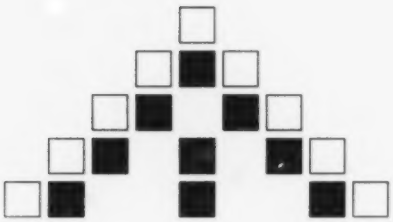

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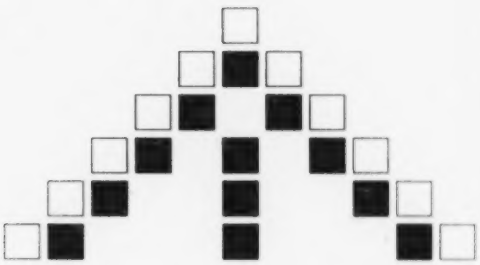

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n

A. The 30th design will use _____ ■s and _____ □s for a total of _____ blocks.

B. Give one way you can find the generalizations for the ■s, the □s, and the total number of blocks by looking at the block arrangements.

_____

_____

C. When we use 96 ■s, we will have a total of _____ blocks.

D. By how much do the numbers grow in each column (Number of Blocks page)?

■s _____ □s _____ Total _____

How does the growing number relate to the generalization for *n* in each column?

_____

_____

E. When we use a total of 128 blocks, _____ will be ■s.

F. When we use 103 □s, we need _____ ■s.

G. Explain how the sum of the generalizations for the ■s and □s equals the generalization for the total number of blocks.

_____

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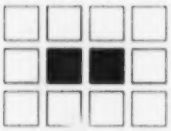
PATTERN 32

1



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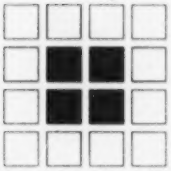

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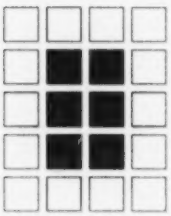

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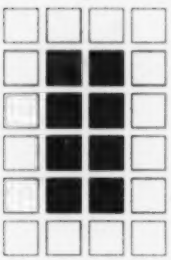

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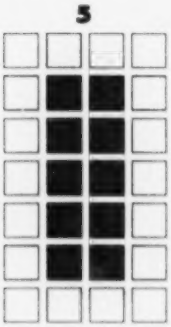

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n

A. The 29th design will use _____ ■s and _____ □s for a total of _____ blocks.

B. Explain one way you can find the generalizations for the ■s, the □s, and the total number of blocks by looking at the block arrangements.

_____

_____

C. When we use 66 ■s, we will use _____ □s.

D. Will the total number of blocks used ever be 255? _____ Why?

_____

256? _____ Why?

_____

E. By how much do the numbers grow in each column (Number of Blocks page)?

■s _____ □s _____ Total _____

How does the growing number relate to the generalization for n in each column?

_____

F. When we have a total of 160 blocks, _____ will be ■s and _____ will be □s.

G. The outside dimensions of the rectangles created in designs 1–5 are shown below. Fill in the missing dimensions.

Design	Base × Height
1	4×3
2	4×4
3	4×5
4	4×6
5	4×7
10	_____
20	_____
n	_____

40

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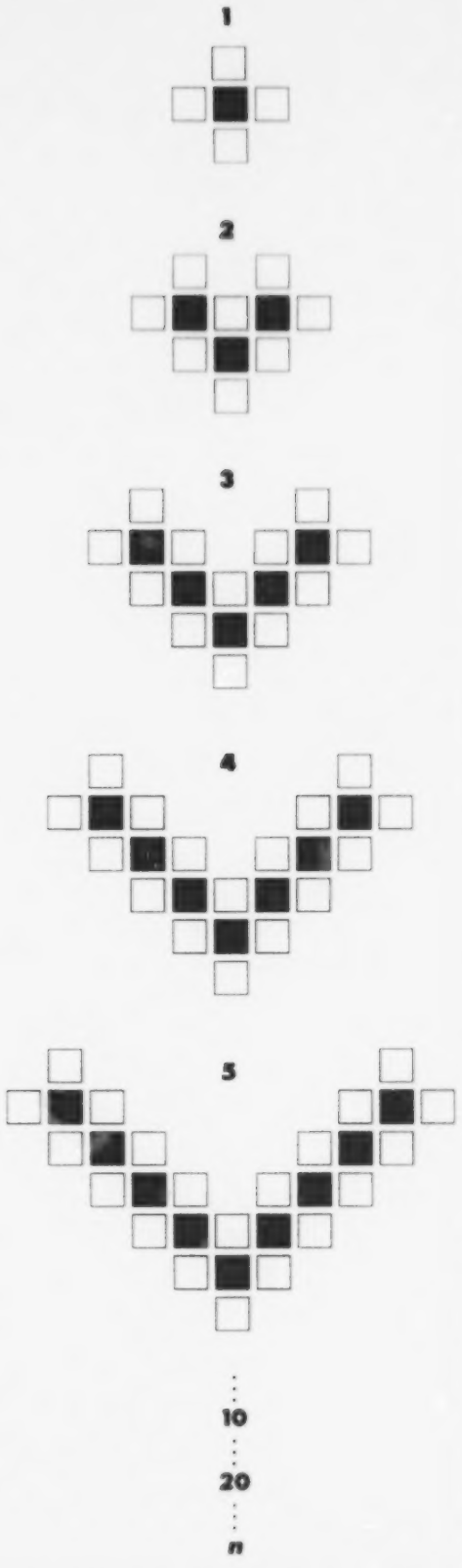
PATTERN 33



A. The 23rd design will use _____ ■s and _____ □s for a total of _____ blocks.

B. Explain one way you can find the generalizations for the ■s, the □s, and the total number of blocks by looking at the physical arrangement of the blocks.

_____

_____

C. When we use 200 □s, we will use _____ ■s.

D. By how much do the numbers grow in each column (Number of Blocks page)?

■s _____ □s _____ Total _____

How does the growing number relate to the generalization in each column?

_____

_____

E. The number of ■s is always odd and the number of □s is always even. Why is the total always odd?

_____

_____

F. When the total is 281 blocks, _____ will be ■s.

G. When we use 139 ■s, there will be _____ □s.

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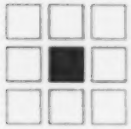
PATTERN 34

1



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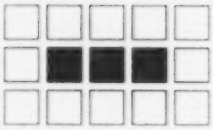

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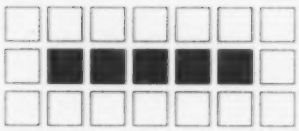

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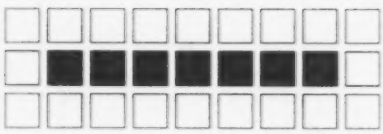

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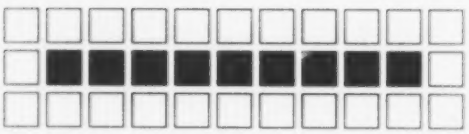

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n

A. The 33rd design will use _____ ■s and _____ □s for a total of _____ blocks.

B. Explain one way you can get the generalization for the ■s, the □s, and the total number of blocks by looking at the block arrangements.

_____

_____

C. Will we ever use an odd number of □s? _____ Why?

_____

_____

Will the total number of blocks ever be an even number? _____ Why?

_____

_____

D. When we use 57 ■s, we will use _____ □s.

E. When the total number of blocks used is 225, there will be _____ ■s and _____ □s.

F. Below are the outside dimensions of the rectangles created in designs 1–5. Fill in the missing dimensions.

Design	Base × Height
1	3 × 3
2	5 × 3
3	7 × 3
4	9 × 3
5	11 × 3
9	_____
10	_____
20	_____
n	_____

42

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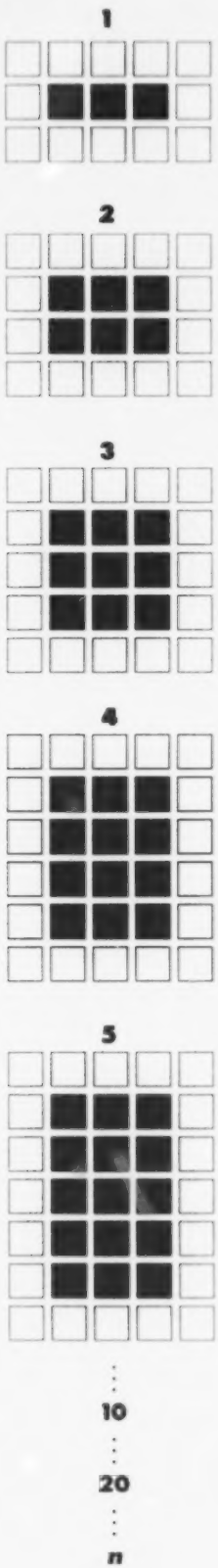
PATTERN 35



A. The 27th design will use _____ ■s and _____ □s for a total of _____ blocks.

B. Explain one way you can get the generalization for the ■s, the □s, and the total number of blocks by looking at the block arrangements.

_____

_____

C. Will we ever use an odd number of ■s ? _____ Why?

_____

of □s? _____ Why?

_____

D. When we use exactly 93 ■s, we will use exactly _____ □s.

E. The total number of blocks used for a design is 210. How many are □s? _____

F. The dimensions of the rectangles created in several designs follow. Fill in the missing dimensions.

Design	Base × Height
1	5 × 3
2	5 × 4
3	5 × 5
4	5 × 6
5	5 × 7
8	_____
10	_____
20	_____
n	_____

G. Explain how the generalization for the total number of blocks equals the sum of the generalizations for the ■s and the □s.

_____

_____

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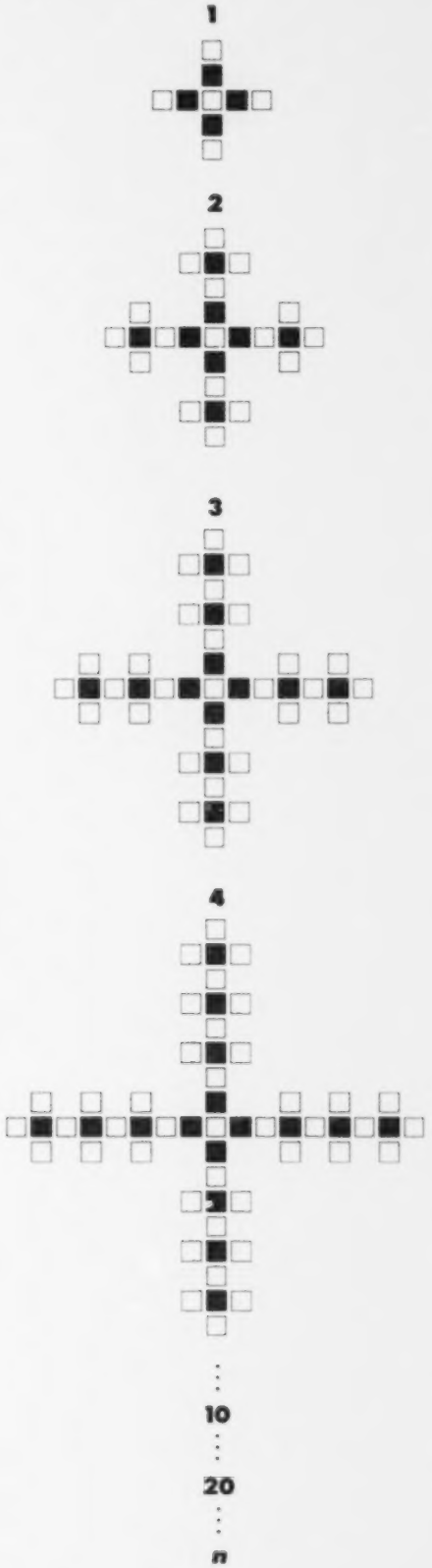
PATTERN 36



A. The 30th design will use _____ ■s and _____ □s for a total of _____ blocks.

B. Explain one way you can find the generalizations for the ■s, the □s, and the total number of blocks by looking at the physical arrangement of the blocks.

_____

_____

C. When we use 56 ■s, we will need _____ □s.

D. Will we ever use exactly 90 ■s to build a design? _____ Why?

_____

_____

E. By how much do the numbers grow in each column (Number of Blocks page)?

■s _____ □s _____ Total _____

How does the growing number relate to the generalization for *n* in each column?

_____

_____

F. When we build the design that requires a total of 633 blocks, _____ will be ■s and _____ will be □s. The next design will use a total of _____ blocks.

G. When we use exactly 293 □s, we will use exactly _____ ■s.

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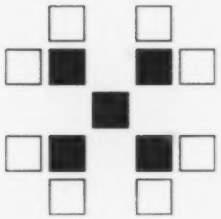
PATTERN 37

1

2



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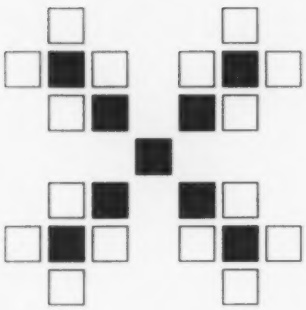

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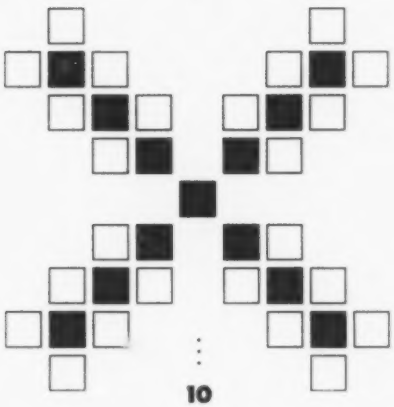

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n

A. The 25th design will use _____ ■s and _____ □s for a total of _____ blocks.

B. Explain one way to find the generalizations for the ■s, the □s, and the total by studying the arrangement of blocks.

_____

_____

C. When we use 117 ■s, we will need _____ □s.

D. By how much do the numbers grow in each column (Number of Blocks page)?

■s _____ □s _____ Total _____

How does the growing number relate to the generalization for *n* in each column?

_____

_____

E. When the total number of blocks is 469, _____ will be ■s and _____ will be □s.

F. Will we ever need an even number of ■s? _____ Why?

_____

Will we ever need an even number for the total? _____ Why?

_____

G. Will we ever use exactly 315 ■s? _____ Why?

_____

H. Explain how the generalization for the total number of blocks equals the sum of the generalizations for the ■s and □s.

_____

_____

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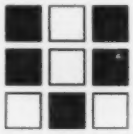
PATTERN 38

1



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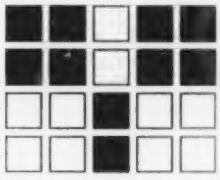

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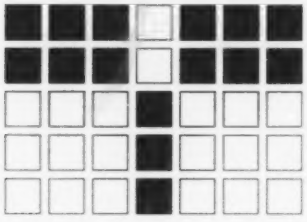

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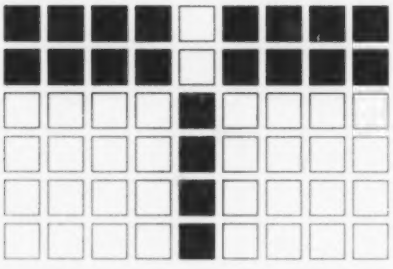

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n

A. The 30th design will use _____ ■s and _____ □s for a total of _____ blocks.

B. Explain one way you can find the generalization for the number of □s by looking at the arrangement of blocks.

_____

C. Will we ever use an odd number of ■s? _____ Why?_____

of □s? _____ Why? _____

_____

D. When we use 75 ■s, we will use_____ □s.

E. Tell whether we will ever use each of the given numbers of blocks and why:

95 ■s _____

95 □s _____

100 □s _____

200 □s _____

F. When we use 130 □s, we will use _____■s.

G. The outside dimensions of several designs follow. Fill in the missing dimensions.

Design	Base × Height
1	3 × 3
2	5 × 4
3	7 × 5
4	9 × 6
5	_____
6	_____
10	_____
20	_____
n	_____

How does the generalization relate to the outside dimensions of the *n*th design?

_____

_____

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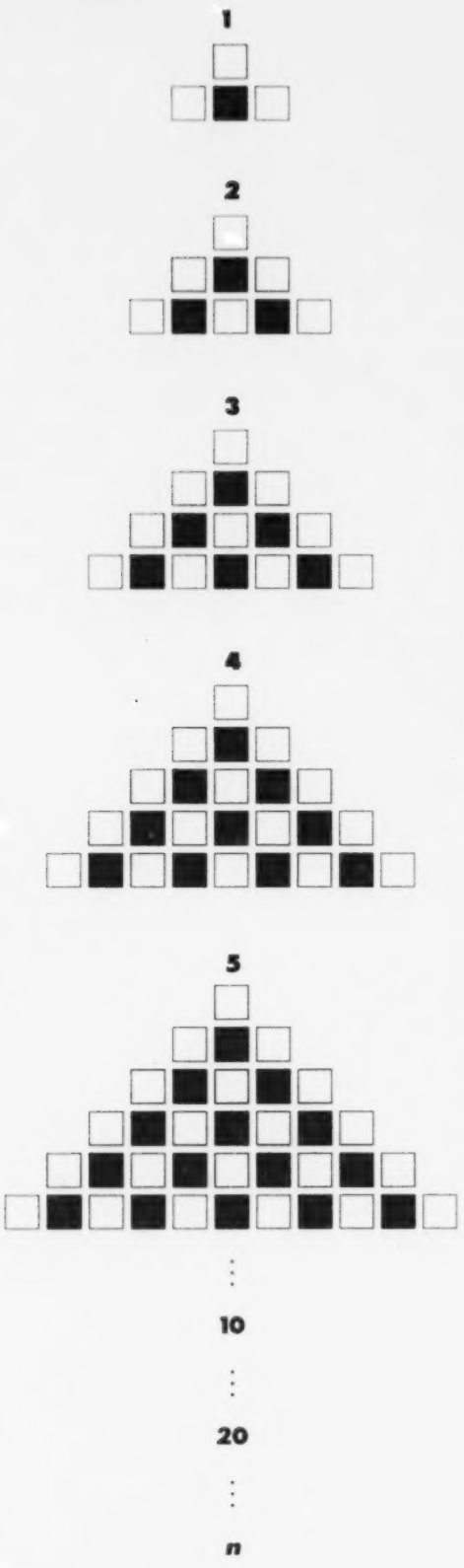
PATTERN 39



A. When you have a total of 2809 blocks, _____ will be ■s and _____ will be □s.

B. Look at the Total column on the Number of Blocks page and describe the pattern created. What do we call these numbers?

_____

_____

C. Describe the number pattern created by the ■s in successive designs.

_____

What are the numbers called?

_____

What geometric shapes do the ■s make?

_____

D. Describe how you can rearrange the blocks in each design to form a square.

_____

The dimensions of several squares are shown below. Fill in the missing ones.

Design	Base × Height
1	2×2
2	3×3
3	4×4
4	5×5
5	6×6
6	_____
7	_____
10	_____
20	_____
n	_____

For any design, how does the generalization for the total number of blocks relate to the dimensions of the square?

_____

_____

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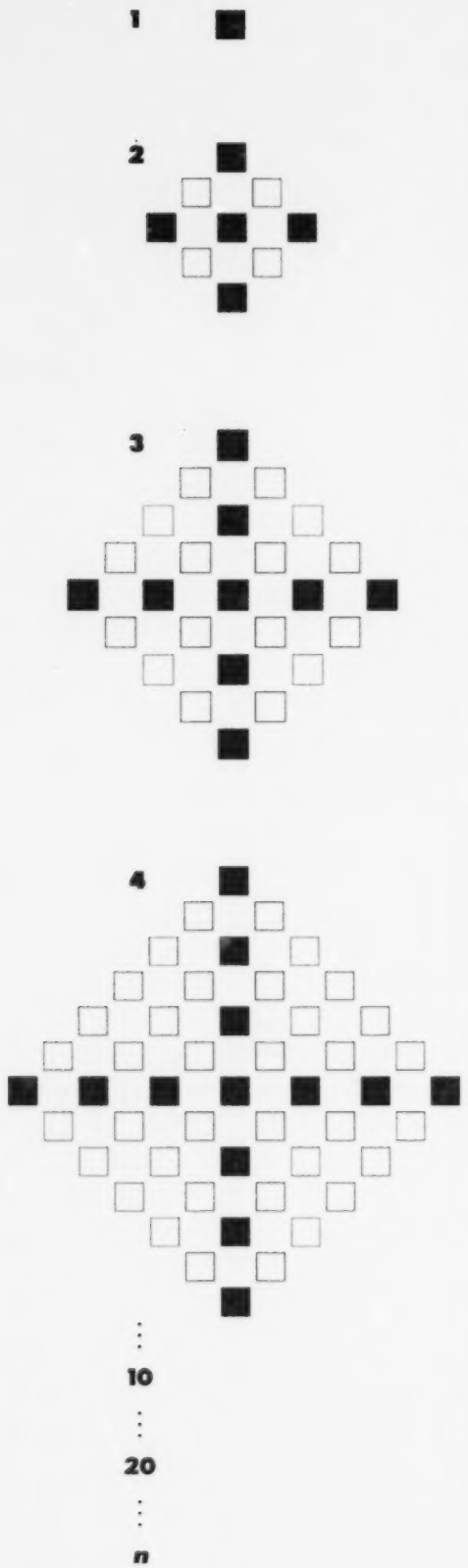
PATTERN 40



A. The 23rd design will use _____ ■s and _____ □s for a total of _____ blocks.

B. By what amounts do the numbers in the ■s column grow?

_____

How does this growing number relate to the generalization for n in that column?

_____

C. Describe how you can rearrange the □s to form a design that is a square.

_____

How can this new arrangement explain the generalization for □?

_____

D. When we use 97 ■s, we will use _____ □s.

E. If we rotate each design 45°, we can think of the designs as squares with the dimensions shown below. Fill in the missing dimensions.

Design	Base × Height
1	1×1
2	3×3
3	5×5
4	7×7
5	9×9
6	_____
10	_____
20	_____
n	_____

How do the dimensions of the square relate to generalization for the Total?

_____

_____

48

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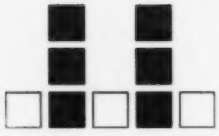
PATTERN 41

1

2



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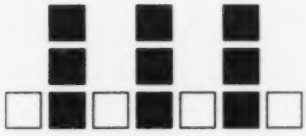

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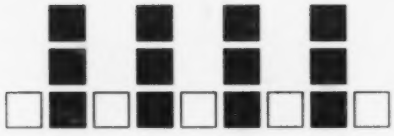

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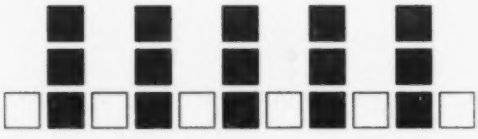

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n

A. The 27th design will use _____ ■s, _____ □s, and _____ ■s for a total of _____ blocks.

B. Explain one way you can, by studying the block arrangements, find the generalizations for the ■s and the total number of blocks.

_____

_____

C. When we use 60 ■s, we will use _____ ■s and _____ □s.

D. By how much do the numbers in the ■s column and in the Total column grow?

■s _____ Total _____

How does the growing number relate to the generalization for *n* in each column?

_____

_____

E. When the total number of blocks is 281, _____ will be ■s, _____ will be □s, and _____ will be ■s.

F. Explain how the generalization for the total number of blocks equals the sum of the generalizations for the ■s, the □s, and the ■s.

_____

_____

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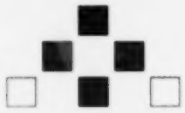
PATTERN 42

1



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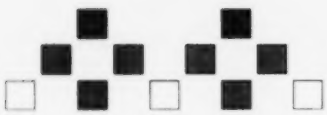

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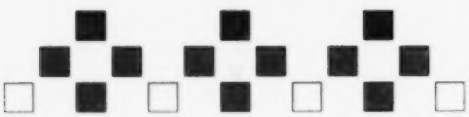

⋮

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⋮

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⋮

n

A. The 35th design will use _____ ■s, _____ □s, and _____ ▩s for a total of _____ blocks.

B. Explain one way the block arrangement shows the generalizations for the ▩s and the total number of blocks.

_____

_____

C. When we use 75 ■s, we will use _____ ▩s and have a total of _____ blocks.

D. By what amounts do the numbers in the ▩s column and the Total column grow?

▩s _____ Total _____

How does the growing number relate to the generalization for *n* in each column?

_____

_____

E. When the total number of blocks is 286, _____ will be ■s, _____ will be □s, and _____ will be ▩s.

F. Tell whether we will ever use each of the given numbers of blocks, and explain why.

103 ▩s _____

103 □s _____

340 blocks _____

341 blocks _____

446 blocks _____

50

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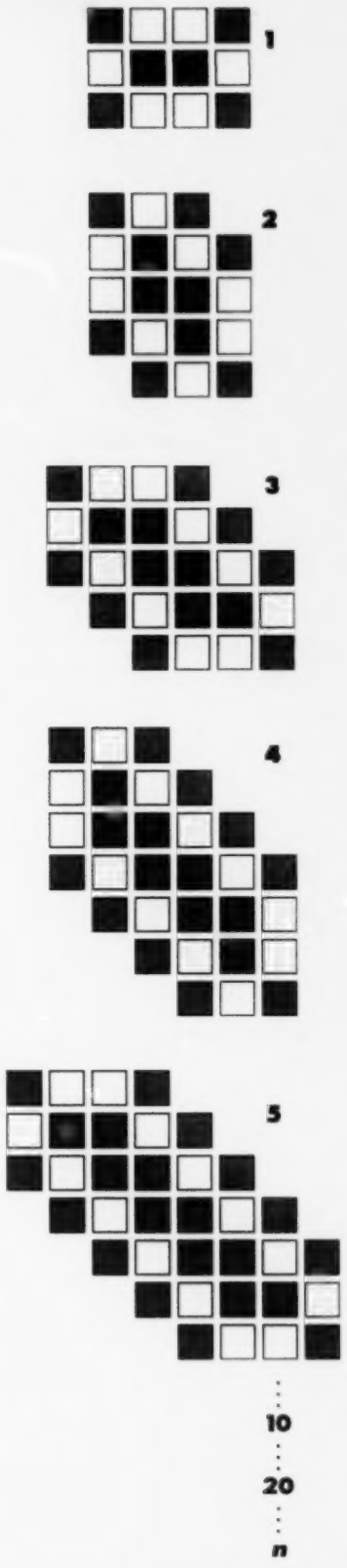
PATTERN 43



A. The 28th design will use _____ ■s, _____ □s, _____ ▩s for a total of _____ blocks.

B. Explain one way you can, by studying the block arrangements, find the generalizations for the □s, the ▩s, and the total number of blocks.

_____

_____

C. When we use 80 ■s, we will use _____ □s and _____ ▩s.

D. By how much do the numbers grow in each column (Number of Blocks page)?

■s _____ □s _____ ▩s _____ Total _____

How does the growing number relate to the generalization for *n* in each column?

_____

_____

E. When the total number of blocks is 486, _____ will be ■s, _____ will be □s, and _____ will be ▩s.

F. What do all numbers in the Total column have in common?

_____

_____

What do we call these numbers?

_____

G. How does the generalization for the Total equal the sum of the generalizations for the ■s, the □s, and the ▩s?

_____

_____

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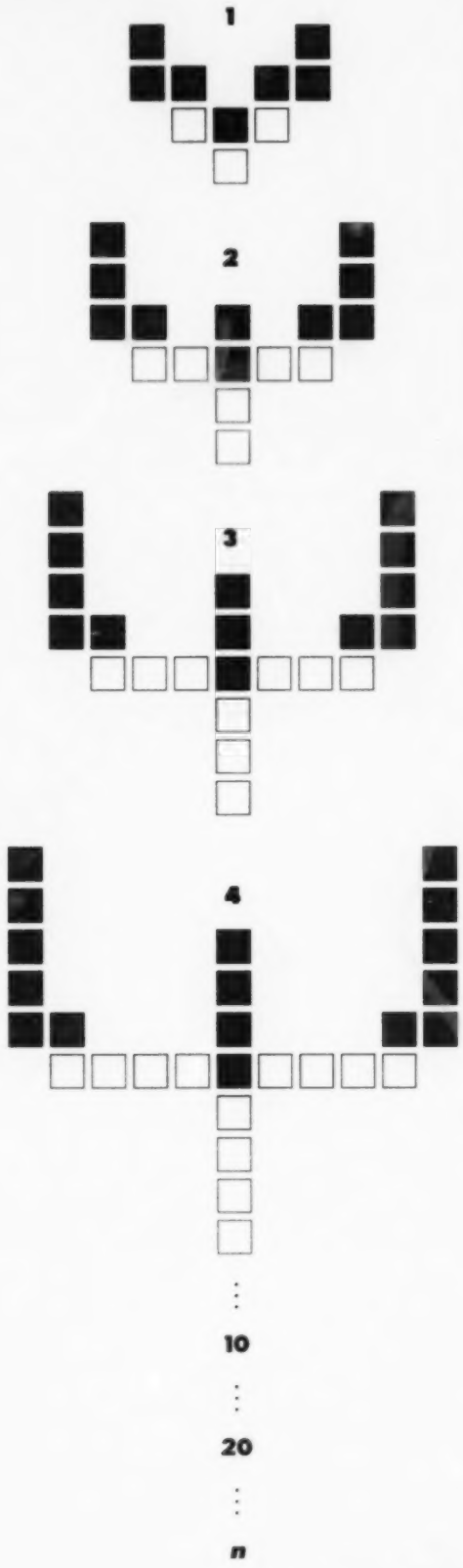
PATTERN 44



A. The 27th design will use _____ ■s, _____ □s, and _____ ▩s for a total of _____ blocks.

B. Explain one way you can, by studying the block arrangements, find the generalizations for the □s, the ▩s, and the total number of blocks.

_____

_____

C. When we use 67 ■s, we will use _____ □s and _____ ▩s.

D. By how much do the numbers grow in each column (Number of Blocks page)?

■s _____ □s _____ ▩s _____ Total _____

How does the growing number relate to the generalization for *n* in each column?

_____

_____

E. When the total number of blocks is 334, _____ will be ■s, _____ will be □s, and _____ will be ▩s.

F. Will we ever use a total of 155 blocks? _____ Why?

_____

160 blocks? _____ Why? _____

_____

G. Explain how the generalization for the Total equals the sum of the generalizations for the ■s, the □s, and the ▩s.

_____

_____

52

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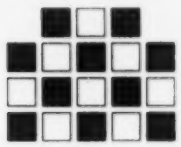
PATTERN 45

1



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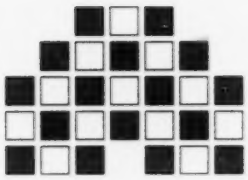

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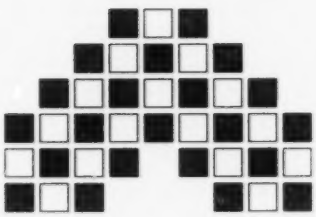

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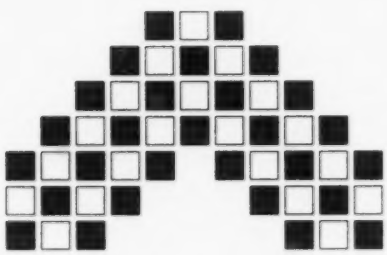

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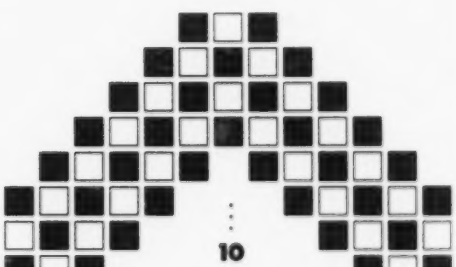

10

20

n

A. The 32nd design will use _____ ■s, _____ □s, and _____ ▪s for a total of _____ blocks.

B. Explain one way you can, by studying the block arrangements, find the generalizations for the ■s, the □s, the ▪s, and the total number of blocks.

_____

_____

C. When we use 81 ■s, we will use _____ □s and _____ ▪s.

D. By how much do the numbers grow in each column (Number of Blocks page)?

■s _____ □s _____ ▪s _____ Total _____

How does the growing number relate to the generalization for *n* in each column?

_____

_____

E. When the total number of blocks is 508, _____ will be ■s, _____ will be □s, and _____ will be ▪s.

F. Tell whether we will ever use each of the given numbers of blocks, and explain why.

80 ■s _____

80 □s _____

80 ▪s _____

80 blocks _____

180 blocks _____

G. Explain how the generalization for the Total equals the sum of the generalizations for the ■s, the □s, and the ▪s.

_____

_____

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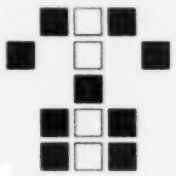
PATTERN 46

1



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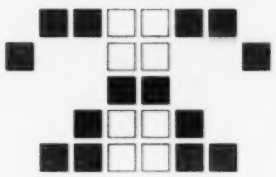

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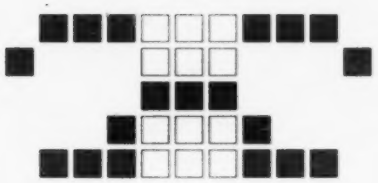

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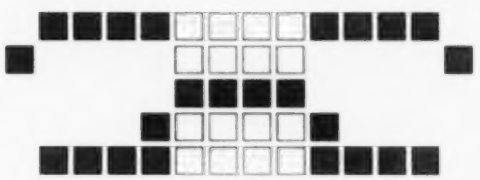

⋮

10

⋮

20

⋮

n

A. The 35th design will use _____ ■s, _____ □s, and _____ ▩s for a total of _____ blocks.

B. Explain one way you can, by studying the block arrangements, find the generalizations for the □s, the ▩s, and the Total.

_____

_____

C. When we use 77 ■s, we will use _____ □s and _____ ▩s.

D. By how much do the numbers grow in each column (Number of Blocks page)?

■s _____ □s _____ ▩s _____ Total _____

How does the growing number relate to the generalization for *n* in each column?

_____

_____

E. When the total number of blocks is 454, _____ will be ■s, _____ will be □s, and _____ will be ▩s.

F. Tell whether we will ever use each of the given numbers of blocks and why.

91 □s _____

112 ▩s _____

112 blocks _____

G. Explain how the generalization for the total number of blocks equals the sum of the generalizations for the ■s, the □s, and the ▩s.

_____

_____

54

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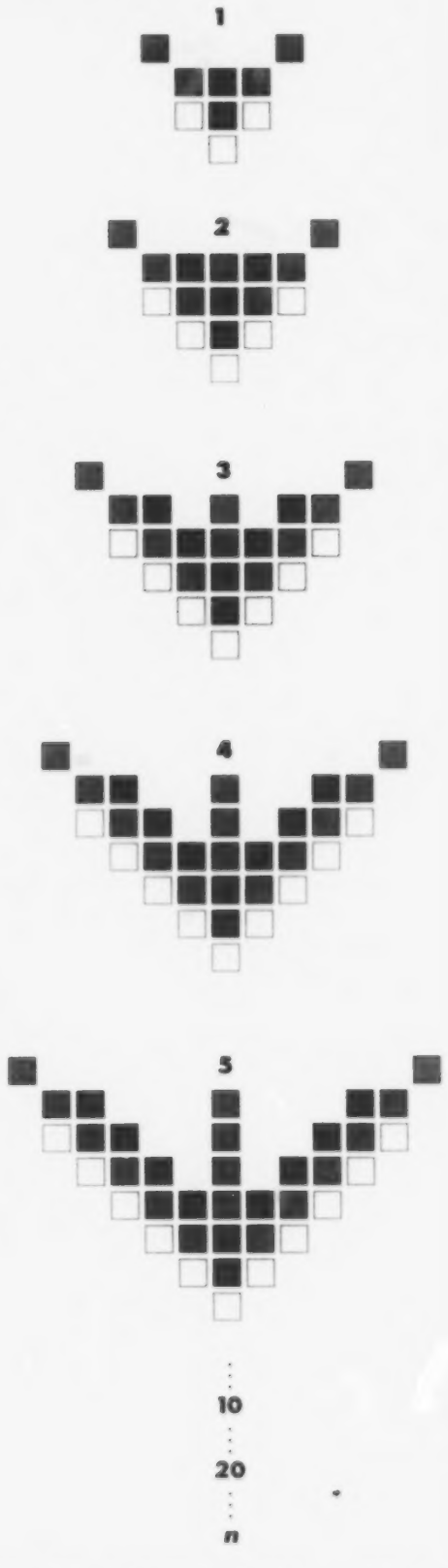
PATTERN 47



A. The 38th design will use _____ ■s, _____ □s, and _____ ▪s for a total of _____ blocks.

B. Explain one way you can, by studying the block arrangements, find the generalizations for the ■s, the □s, the ▪s and the total number of blocks.

_____

_____

C. When we use 96 ■s, we will use _____ □s and _____ ▪s.

D. By how much do the numbers grow in each column (Number of Blocks page)?

■s _____ □s _____ ▪s _____ Total _____

How does the growing number relate to the generalization for n in each column?

_____

_____

E. When the total number of blocks is 422, _____ will be ■s, _____ will be □s, and _____ will be ▪s.

F. Tell whether we will ever use each of the given numbers of blocks and why.

115 ■s _____

115 □s _____

115 ▪s _____

310 blocks _____

G. Explain how the generalization for the total number of blocks equals the sum of the generalizations for the ■s, the □s, and the ▪s.

_____

_____

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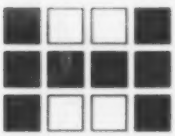
PATTERN 48

1



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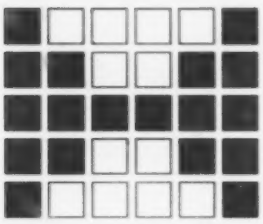

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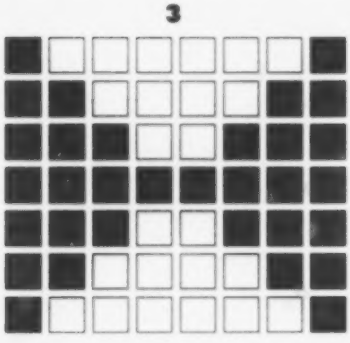

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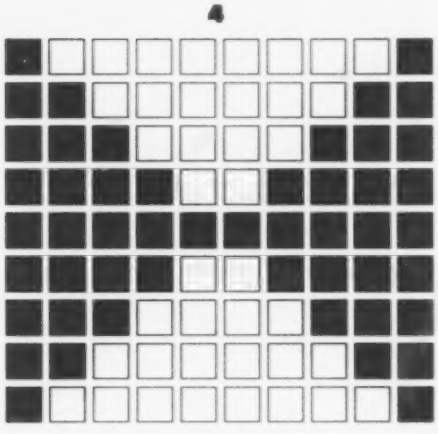

⋮

10

⋮

20

⋮

n

A. The 34th design will use _____ ■s, _____ □s and, _____ ■s for a total of _____ blocks.

B. Explain one way you can, by studying the block arrangements, find the generalizations for the ■s and the ■s.

_____

_____

C. When we use 102 ■s, we will use _____ □s and _____ ■s.

D. Tell whether we will ever use each of the given numbers of blocks and why.

124 ■s _____

288 □s _____

288 ■s _____

870 blocks _____

E. Below are the outside dimensions of the rectangles created in designs 1–5. Fill in the missing dimensions.

Design	Base × Height
1	4×3
2	6×5
3	8×7
4	10×9
5	_____
10	_____
20	_____
n	_____

F. Explain the relationship between the Total generalization and the dimensions of the rectangle for the *n*th design.

_____

_____

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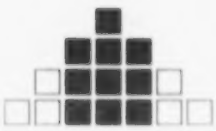
PATTERN 49

1

2



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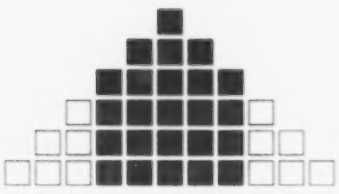

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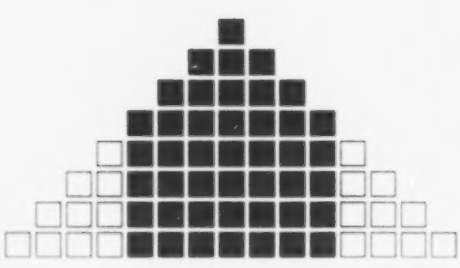

⋮

10

⋮

20

⋮

n

A. The 35th design will use _____ ■s, _____ □s, and _____ ■s for a total of _____ blocks.

B. Explain one way you can find the generalizations for the ■s and the number of □s by studying the block arrangements.

_____

_____

C. When we use 324 ■s, we will use _____ □s and _____ ■s for a total of _____ blocks.

D. The dimensions of the rectangles formed by several ■ designs are shown below. Fill in the missing dimensions.

Design	Base × Height
1	1×1
2	3×2
3	5×3
4	7×4
5	_____
10	_____
20	_____
n	_____

How do the dimensions of the *n*th rectangle relate to the generalization for the number of ■s?

_____

_____

E. Explain how you can rearrange each design into a square.

_____

_____

How does the square arrangement show the generalization for the total number of blocks?

_____

57 _____

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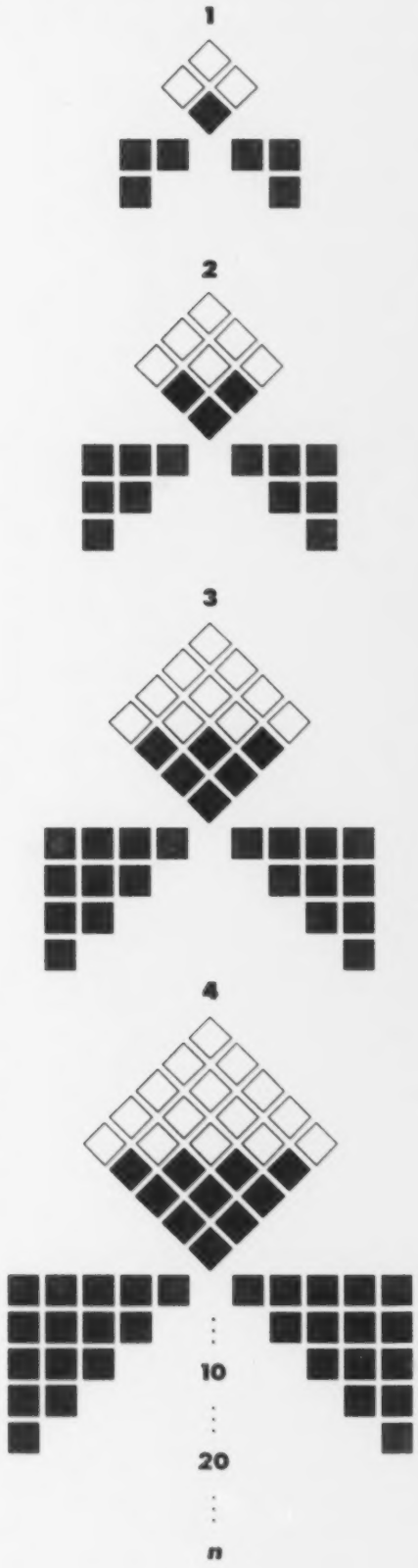
PATTERN 50



A. The 37th design will use _____ ■s, _____ □s, and _____ ▩s for a total of _____ blocks.

B. The ■s and □s together make a square that increases in size as the pattern progresses. Fill in the blanks below with the correct dimensions for the square of combined ■s and □s.

Design	Base × Height
1	2×2
2	3×3
3	4×4
4	5×5
5	_____
10	_____
20	_____
n	_____

Explain how the generalizations for the ■s and □s, when added together, give us the above dimensions.

_____

_____

C. When we use 528 ■s, we will use _____ □s and _____ ▩s for a total of _____ blocks.

D. Explain one way you can find the generalizations for each of the following blocks by studying their arrangements.

▩s: _____

■s: _____

E. Explain how the generalization for the total number of blocks equals the sum of the generalizations for the ■s, the □s, and the ▩s.

_____

_____

58

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APPENDIX

Glossary of Terms

Building Algebraic Thinking uses a specific vocabulary in providing valuable pre-algebra experiences to the student; it is important that the teacher become familiar with the terms used in the book.

coefficient of *n*: number by which the variable *n* is multiplied

constant difference (growing number): the difference in numbers of blocks used between any two consecutive designs of a pattern

generalization: general equation describing the relationship between the ordinal number of a design (*n*) and the number of blocks used for that design; the "rule" or formula for a pattern

linear function: generalization whose solutions for individual designs increase by a consistent number and whose equation can be plotted as a straight line

pattern: a series of an infinite number of consistent designs in which there is one possible arrangement and number of blocks for each design; the pattern has a "rule" or generalized equation that can be used to determine the numbers of blocks needed in any design

design: arrangement of blocks that represents a stage or progression within a pattern (each activity page shows several designs of one overall pattern); each design requires a specific number and arrangement of blocks consistent with the pattern

quadratic function: generalization involving the square of the variable; it will take on the general form of $ax^2 + bx + c$; graph of this function is a curved line (parabola)

variable: a symbol used as a place holder for a number that can vary in value

reverse processing: solving an equation in which the unknown (variable) is on the left side: instead of finding the number of blocks used in a given pattern design ($2 \times 5 = ?$), the student is given the number of blocks required and must determine the associated design ($2 \times ? = 10$) or the associated number of another kind of block in the design

Teaching Guide

Building Algebraic Thinking is intended to be an exploratory/discovery based program; a formalistic teaching approach is not recommended. However, for those who will gain confidence with some initial structure, the guidelines below provide a questioning strategy that can be used as a model to get you started. Review them before teaching, and as you become familiar with the concepts, rely on your own judgment of what your students need moment-by-moment.

The order of activities for any pattern can be altered as needed: the student may copy and extend the pattern for several designs, fill in the associated numbers of blocks, and answer one or more questions before completing every step in the guide. As you proceed, keep in mind the following points:

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- The steps will familiarize you with the general concepts for the activity pages. In providing problems to the students, you may vary the types and extent of activities on each page; the important thing is that students progress, when they are ready, from the simple concrete experiences to the abstract concepts that naturally evolve.

- Students may need to just experiment and build many patterns before they are ready to formalize the concepts elicited through any questioning strategy. Be careful not to push them into a stage for which they are not yet ready. It is important for them to get comfortable with each new realization; in the process, don't worry if they seem to "regress" into a previous stage of simple block building.

1. Building the designs

Have the students use their blocks to copy the exact arrangement of objects for the designs shown on the activity page. As they proceed, ask questions to stimulate their analysis of the pattern:

What does this design resemble? How is it growing? What will happen as it grows?

2. Determining the numbers of blocks

Ask students to notice the numbers of blocks used for each pattern design and fill in the blank(s) of the second column on the Number of Blocks page (p. 54).

How many blocks will we need for the first design? the second? the fifth?

3. Discovering the pattern

Allow the students to discover any number pattern occurring on the Number of Blocks page they have filled out (including odd or even numbers) and the relationship between the design number and the number of blocks used.

How are the numbers of blocks used different from one design to the next? Do you notice any kind of pattern developing in the number of blocks column? How many blocks do we add to each design? How do these "growing" numbers change? Are the differences always the same? Are they odd or even?

When the student can easily see how the numbers change regularly, you may want to use the term **constant difference**.

As the numbers grow, what is the difference occurring between designs? Is there a constant difference—is it the same difference from one design to the next throughout the pattern?

4. Extending the pattern

Encourage the students to "work forward" by having them predict the number of blocks needed for subsequent designs, and then encourage reverse processing by having students predict what design number will require a given number of blocks.

Can you predict how many blocks will be needed in order to build the sixth and seventh designs?

How did you figure out the solution? How is the number of blocks used related to the number of the design?

Which design will require a total of 86 blocks? With 200 blocks, which design could you build?

When the students are able to predict the answers for specific designs, encourage them to explain how they got their answers.

How did you know how many blocks to use for design number 7? What was your formula? [e.g., 2 times the design number or 2×7]

Encourage the students to use mental arithmetic as they determine the rule for generalizing; let different students give the design number and the rest of the class give the end result.

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Are there any numbers that will stump us? Can we find the answer for any whole number?

When students can easily give the number of blocks, use *n* as a variable to represent any design number.

5. Generalizing with a variable

Encourage students to use a variable to create a generalization that can be used to determine the number of blocks for any design in the pattern.

Is there an easy way to find the answer for any design of this particular pattern?

You may want to start using the term *generalize.*

You've already come up with some specific formulas for the different designs. How about a general formula that applies to the whole pattern? What if we let n *stand for any number? What do we do to* n*?* (The same as we did to the design number: $2 \times n$ = number of blocks.)

Have students substitute any number for *n*.

Now, how can we find the number of blocks we need to build the 20th design? What do we do with the number 20? (Substitute for *n*: $2 \times 20 = 40$ blocks.)

When the final generalization is found, it should be checked. Have students go back to the earlier, simpler designs and substitute design numbers for the variable to show that the generalization does indeed work.

6. Analyzing the physical arrangement

It is hoped that students will see that there are many different ways to solve problems and that mathematics is not a one-right-answer discipline. Students may come up with some very creative strategies, and these should be shared.

We have seen that constant difference may help students generalize. Another strategy they may find helpful is to analyze the physical arrangement of blocks.

Explain one way you can see the final generalization for n *by looking at the physical arrangement of blocks. How does the block arrangement show that the final generalization is true?* (Students may explore to discover the connection; for a generalization that includes $5n$, for example, they might look for 5 groups of *n* blocks in the physical arrangement. The groups may be in rows at first and later in more obscure cluster arrangements that must be observed carefully.)

What do you notice about the dimensions of the arrangement? Do you see any relationship to the generalization for n*?*

As the designs become more complex, it can become difficult to see the generalization in the physical arrangement, and the students may rely on the constant difference method. Whatever works for the student is fine.

Reproducible "Number of Blocks" Pages

The Number of Blocks pages (pp. 54–56) are to be reproduced and distributed so that students always have the correct sheet for the section they are working on. As the students build the block designs, they record in the blanks of the Number of Blocks sheet the numbers of blocks used for each design. Use the chart at right as a quick reference for specific information on the Number of Blocks pages.

Number of Blocks Page	Types of Blocks	Corresponding Patterns
Section 1 — p. 54	White blocks	Patterns 1–20
Section 2 — p. 55	White and black blocks	Patterns 21–40
Section 3 — p. 56	White, black, and gray blocks	Patterns 41–50

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Section 1

NUMBER OF BLOCKS

PATTERN _____

Design	□s
1	
2	
3	
4	
5	
• • •	• • •
10	
• • •	• • •
20	
• • •	• • •
n	

PATTERN _____

Design	□s
1	
2	
3	
4	
5	
• • •	• • •
10	
• • •	• • •
20	
• • •	• • •
n	

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Section 2
NUMBER OF BLOCKS

PATTERN _____

Design	■s	□s	Total
1			
2			
3			
4			
5			
• • •	• • •	• • •	• • •
10			
• • •	• • •	• • •	• • •
20			
• • •	• • •	• • •	• • •
n			

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Section 3
NUMBER OF BLOCKS

PATTERN _____

Design	■s	□s	■s	Total
1				
2				
3				
4				
5				
• • •	• • •	• • •	• • •	• • •
10				
• • •	• • •	• • •	• • •	• • •
20				
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ANSWER KEY

Note: The explanations included in the answer key are intended for the teacher (for clarification on how the answer *could* be found). It is not suggested that the students be given explanations or specific directions on how to find answers; students should be allowed to discover their own unique methods for solving the problems.

SECTION 1

Pattern 1 Answers

A. 32
If $n = 16$, then $2n = 32$ □s.

B. We add 2 to each successive number, resulting in the even numbers or multiples of 2.

C. No. We only use even numbers of blocks (multiples of 2); 79 is an odd number.

D. 134
If $n = 67$, then $2n = 134$ □s.

E. 42nd
If $2n = 84$ □s, then $n = 42$.

F. 2×8
2×10
2×20
$2 \times n$

Design	□s
1	2
2	4
3	6
4	8
5	10
⋮	⋮
10	20
⋮	⋮
20	40
⋮	⋮
n	n + n or 2n

Pattern 2 Answers

A. 51
If $n = 17$, then $3n = 51$ □s.

B. We add 3 to each successive number, resulting in the multiples of 3.

C. No; 91 is not a multiple of 3.

D. 9th
If $3n = 27$ □s, then $n = 9$.

E. 177
If $n = 59$, then $3n = 177$ □s.

F. 3×9
3×10
3×20
$3 \times n$

Design	□s
1	3
2	6
3	9
4	12
5	15
⋮	⋮
10	30
⋮	⋮
20	60
⋮	⋮
n	n + n + n or 3n

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Pattern 3 Answers

A. 30
If $n = 15$, then $2n = 30$ □s.

B. We add 2 to each number, resulting in the even numbers or multiples of 2.

C. 25th
If $2n = 50$ □s, then $n = 25$.

D. No; 65 is not an even number, or multiple of 2.

E. 178
If $n = 89$, then $2n = 178$ □s.

F. 189th
If $2n = 378$ □s, then $n = 189$.

G. The designs on page 1 use the same numbers of blocks as the designs on this page. The blocks are arranged differently. One way of rearranging the □s on this page so that they are the same as those on page 1 is by moving the two □s in the middle, one to the top of each column, and pushing the columns together.

Design	□s
1	2
2	4
3	6
4	8
5	10
⋮	⋮
10	20
⋮	⋮
20	40
⋮	⋮
n	n + n or 2n

Pattern 4 Answers

A. 95
If $n = 19$, then $5n = 95$ □s.

B. We add 5 to each number, resulting in the multiples of 5.

C. 5
The 5 is the number in front of the n in the generalization $5n$.

D. 38th
If $5n = 190$ □s, then $n = 38$.

E. No; 224 is not a multiple of 5.

F. 5×8
5×10
5×20
$5 \times n$

Design	□s
1	5
2	10
3	15
4	20
5	25
⋮	⋮
10	50
⋮	⋮
20	100
⋮	⋮
n	5n

Pattern 5 Answers

A. 39
If $n = 13$, then $3n = 39$ □s.

B. The numbers increase by 3; the resulting numbers are multiples of 3.

C. 3
The 3 is the number in front of the n in the generalization $3n$.

D. 30th
If $3n = 90$ □s, then $n = 30$.

E. No; 35 is not a multiple of 3.

F. 101st
If $3n = 303$ □s, then $n = 101$.

G. The □s on page 5 can be rearranged into 3 columns with n □s in each.

Design	□s
1	3
2	6
3	9
4	12
5	15
⋮	⋮
10	30
⋮	⋮
20	60
⋮	⋮
n	3n

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Pattern 6 Answers

A. 72
If $n = 18$, then $4n = 72$ □s.

B. The numbers increase by 4. These numbers are the multiples of 4.

C. No. Multiples of 4 are always even numbers.

D. 32nd
If $4n = 128$ □s, then $n = 32$.

E. 236
If $n = 59$, then $4n = 236$ □s.

F. 4×8
4×10
4×20
$4 \times n$

Design	□s
1	4
2	8
3	12
4	16
5	20
⋮	⋮
10	40
⋮	⋮
20	80
⋮	⋮
n	4n

Pattern 7 Answers

A. 75
If $n = 15$, then $5n = 75$ □s.

B. There are always 5 groups of blocks: three columns of n and 2 "wings" of n each at the bottom: $3n + 2n = 5n$.

C. No; 42 is not a multiple of 5.

D. 5. The 5 is the number in front of the n in the generalization $5n$.

E. 19th
If $5n = 95$ □s, then $n = 19$.

F. 400
If $n = 80$, then $5n = 400$ □s.

G. 117th
If $5n = 585$ □s, then $n = 117$.

H. If we move the bottom "wings" of each design on this page up into two new columns (one on each side of the existing columns), we have the same design as on page 4.

Design	□s
1	5
2	10
3	15
4	20
5	25
⋮	⋮
10	50
⋮	⋮
20	100
⋮	⋮
n	5n

Pattern 8 Answers

A. 35
If $n = 18$, then $2n - 1 = 35$ □s.

B. Starting with 1, we add 2 to each number and get the odd numbers.

C. No; 56 is not an odd number.

D. 25th
If $2n - 1 = 49$ □s, then $n = 25$.

E. No. We need an even number of □s to form a rectangle.

F. 129
If $n = 65$, then $2n - 1 = 129$ □s.

G. 266th
If $2n - 1 = 531$ □s, then $n = 266$.

H. The blocks are always in 2 rows with n blocks in the bottom row and $n - 1$ blocks in the top row. This gives $n + (n - 1) = (2n - 1)$.

Design	□s
1	1
2	3
3	5
4	7
5	9
⋮	⋮
10	19
⋮	⋮
20	39
⋮	⋮
n	2n – 1

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Pattern 9 Answers

Design	□s
1	2
2	5
3	8
4	11
5	14
⋮	⋮
10	29
⋮	⋮
20	59
⋮	⋮
n	3n – 1

A. 44
If $n = 15$, then $3n - 1 = 44$ □s.

B. 3. The 3 is the number in front of n in our generalization of $3n - 1$.

C. Odd (119 □s). Even design numbers use an odd number of blocks.

D. 50th
If $3n - 1 = 149$ □s, then $n = 50$.

E. 7×3
9×3
19×3
$(n - 1) \times 3$
If you use n, the dimensions of the inside rectangle can be written as $(n - 1) \times 3$, or $(3n - 3)$. Adding the 2 extra blocks, you get $(3n - 3) + 2 = (3n - 1)$.

Pattern 10 Answers

Design	□s
1	5
2	7
3	9
4	11
5	13
⋮	⋮
10	23
⋮	⋮
20	43
⋮	⋮
n	2n + 3

A. 47
If $n = 22$, then $2n + 3 = 47$ □s.

B. 2. The 2 is the number in front of n in our generalization of $(2n + 3)$.

C. There are a constant 3 blocks in the center and two arms on each side. Each arm has n blocks so the total is $2n + 3$.

D. 100th
If $2n + 3 = 203$ □s, then $n = 100$.

E. 133
If $n = 65$, then $2n + 3 = 133$ □s.

F. In both patterns, the number of blocks used in each design is increased by adding 2 to the previous number. The number of blocks used in each design from Pattern 10 is always 4 more than the number of blocks used in the design from Pattern 8. The generalization for Pattern 10 is 4 more than the generalization for Pattern 8: $(2n + 3)$ is 4 more than $(2n - 1)$.

Pattern 11 Answers

Design	□s
1	1
2	3
3	5
4	7
5	9
⋮	⋮
10	19
⋮	⋮
20	39
⋮	⋮
n	2n – 1

A. 25
If $n = 13$, then $2n - 1 = 25$ □s.

B. Starting with 1, we add 2 each time. These are the odd numbers.

C. 15th
If $2n - 1 = 29$ □s, then $n = 15$.

D. 99
If $n = 50$, then $2n - 1 = 99$ □s.

E. 100th
If $2n - 1 = 199$ □s, then $n = 100$.

F. The numbers on page 11 are always 1 less than the corresponding numbers on page 1. The generalization on this page $(2n - 1)$ is 1 less than the generalization for page 1 $(2n)$.

G. The blocks are laid out in the shape of a W, of which each arm has $n - 1$ blocks; there is always one constant block in the middle. Therefore, the total will be

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$2(n - 1) + 1$. Multiply each term in parentheses by 2 and add the 1 to get $(2n - 2) + 1 = 2n - 1$.

Pattern 12 Answers

A. 66
If $n = 17$, then $4n - 2 = 66$ □s.
B. Starting with the number 2, we add 4 each time.
C. No; 89 is an odd number, and we are only using even numbers of □s.
D. 4. The 4 is the number in front of n in generalization $4n - 2$.
E. 90th.
If $4n - 2 = 358$ □s, then $n = 90$.
F. 17×2
19×2
39×2
$(2n - 1) \times 2 = 4n - 2$

Design	□s
1	2
2	6
3	10
4	14
5	18
⋮	⋮
10	38
⋮	⋮
20	78
⋮	⋮
n	4n – 2

Pattern 13 Answers

A. 41
If $n = 14$, then $3n - 1 = 41$ □s.
B. 3. The 3 is the number in front of n in our generalization of $3n - 1$.
C. 25th
If $3n - 1 = 74$ □s, then $n = 25$.
D. The blocks are always arranged as a T with a constant two blocks on top. Each arm of the T has $n - 1$ blocks so the total is $3(n - 1) + 2 = (3n - 3) + 2 = 3n - 1$.
E. 209
If $n = 70$, then $3n - 1 = 209$ □s.
F. 40th
If $3n - 1 = 119$ □s, then $n = 40$.
G. The numbers for page 13 are always one less than those for page 2. The generalization for page 13 is also one less: $3n - 1$ compared to $3n$.

Design	□s
1	2
2	5
3	8
4	11
5	14
⋮	⋮
10	29
⋮	⋮
20	59
⋮	⋮
n	3n – 1

Pattern 14 Answers

A. 35
If $n = 16$, then $2n + 3 = 35$ □s.
B. Starting with the number 5, we add 2 each time.
C. There are always 3 blocks in the middle row and n blocks above and n blocks below the middle row for a total of $2n + 3$.
D. 103
If $n = 50$, then $2n + 3 = 103$ □s.
E. 46th If $2n + 3 = 95$ □s, then $n = 46$.
F. 100th
If $2n + 3 = 203$ □s, then $n = 100$.
G. The numbers for this page are always 4 more than the ones for page 8. The generalization is also 4 more: $2n + 3$ compared to $2n - 1$.

Design	□s
1	5
2	7
3	9
4	11
5	13
⋮	⋮
10	23
⋮	⋮
20	43
⋮	⋮
n	2n + 3

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Pattern 15 Answers

A. 70
If $n = 23$, then $3n + 1 = 70$ □s.

B. 3. The 3 is the number in front of n in our generalization of $3n + 1$.

C. The blocks are always in 3 columns of n blocks each. There is also the constant 1 block on top. The total is therefore $3n + 1$.

D. 40th
If $3n + 1 = 121$ □s, then $n = 40$.

E. 250
If $n = 83$, then $3n + 1 = 250$.

F. The numbers for this page are always 1 more than the ones for page 5. The generalization is also 1 more: $3n + 1$ compared to $3n$.

Design	□s
1	4
2	7
3	10
4	13
5	16
⋮	⋮
10	31
⋮	⋮
20	61
⋮	⋮
n	3n + 1

Pattern 16 Answers

A. 53
If $n = 14$, then $4n - 3 = 53$ □s.

B. The blocks all lie along 4 diagonals; each diagonal has $n - 1$ blocks. There is also the constant block in the center, so the total is $4(n - 1) + 1 = (4n - 4) + 1 = 4n - 3$. This is the generalization for n.

C. 4. The 4 is the number in front of the n in the generalization $4n - 3$.

D. 22nd
If $4n - 3 = 85$ □s, then $n = 22$.

E. No. No whole number n satisfies $4n - 3 = 50$. We use 49 □s for the 13th step or 53 □s for the 14th. The number of □s will always be odd.

F. 317
If $n = 80$, then $4n - 3 = 317$ □s.

G. 50th
If $4n - 3 = 197$ □s, then $n = 50$.

H. The numbers for page 16 are always 3 less than the numbers for page 6. The generalization is also 3 less: $4n - 3$ compared to $4n$.

Design	□s
1	1
2	5
3	9
4	13
5	17
⋮	⋮
10	37
⋮	⋮
20	77
⋮	⋮
n	4n - 3

Pattern 17 Answers

A. 49
If $n = 15$, then $3n + 4 = 49$ □s.

B. 3. The 3 is the number in front of n in our generalization of $3n + 4$.

C. 25th
If $3n + 4 = 79$ □s, then $n = 25$.

D. The blocks are always in the shape of a Y with a constant block at the junction of the arms. Each of the three arms always has $n + 1$ blocks, so the total will be $3(n + 1) + 1 = (3n + 3) + 1 = 3n + 4$. We can also look at each design as having a constant core of 4 at the junction; each arm has n, so the total number of blocks is $3n + 4$.

E. 154
If $n = 50$, then $3n + 4 = 154$.

F. 100th
If $3n + 4 = 304$, then $n = 100$.

Design	□s
1	7
2	10
3	13
4	16
5	19
⋮	⋮
10	34
⋮	⋮
20	64
⋮	⋮
n	3n + 4

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G. The numbers for this page are always 5 more than the numbers for page 9. The generalization for this page is also 5 more: $3n + 4$ compared to $3n - 1$.

Pattern 18 Answers

A. 34
If $n = 18$, then $2n - 2 = 34$ □s.

B. Starting with the number 0, we add 2 to each number. These are the even numbers (multiples of 2).

C. The blocks are always in two rows, each row with $n - 1$ blocks. The total is therefore $2(n - 1) = 2n - 2$.

D. 2. The 2 is the number in front of n in the generalization $2n - 2$.

E. 100th
If $2n - 2 = 198$ □s, then $n = 100$.

F. The numbers on this page are always 2 less than the numbers for page 1. The generalization is also 2 less: $2n - 2$ compared to $2n$.

Design	□s
1	0
2	2
3	4
4	6
5	8
⋮	⋮
10	18
⋮	⋮
20	38
⋮	⋮
n	2(n – 1) or 2n – 2

Pattern 19 Answers

A. 73
If $n = 13$, then $6n - 5 = 73$ □s.

B. 6. The 6 is the number in front of n in our generalization of $6n - 5$.

C. There are 4 arms plus a block in the middle. The lower two arms each have $n - 1$ blocks and the upper two arms each have $2n - 2$. The total is $2(n - 1) + 2(2n - 2) + 1 = 2n - 2 + 4n - 4 + 1 = 6n - 5$. Multiply the terms in parentheses by the coefficient; combine "like terms."

D. 295
If $n = 50$, then $6n - 5 = 295$ □s.

E. 40th
If $6n - 5 = 235$ □s, then $n = 40$.

F. No; $6n$ will be even, but we subtract 5 and get an odd number.

G. 83rd
If $6n - 5 = 493$ □s, then $n = 83$.

Design	□s
1	1
2	7
3	13
4	19
5	25
⋮	⋮
10	55
⋮	⋮
20	115
⋮	⋮
n	6n – 5

Pattern 20 Answers

A. 256
If $n = 16$, then $n^2 = 256$ □s.

B. To the blocks of each design, we add successive odd numbers (3, 5, 7, 9, 11, etc.) of blocks. In other words, we add $2n + 1$ blocks to each new n.

C. Successive odd numbers are added at each step, resulting in a pattern made of the square numbers. They are called square numbers because the block design for each step can be made into the shape of a square.

D. 12th
If $n^2 = 144$ □s, then $n = 12$.

E. No; 80 is not a square number.

F. 25th
If $n^2 = 625$ □s, then $n = 25$.

Design	□s
1	1
2	4
3	9
4	16
5	25
⋮	⋮
10	100
⋮	⋮
20	400
⋮	⋮
n	n^2

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G. You can take the first $n - 1$ columns, rotate them counterclockwise 90°, and fit them into the remaining blocks to form a square. The square has dimensions of $n \times n = n^2$, which is our final answer.

SECTION 2

Pattern 21 Answers

A. 50, 75, 125
If $n = 25$, then $2n = 50$ ■s, $3n = 75$ □s, and $5n = 125$ blocks.

B. The ■s are always arranged in 2 columns with n blocks in each column for a total of $2n$ ■s. The □s are always in 3 columns with n blocks in each column for a total of $3n$ □s.

C. 150
If $2n = 60$ ■s, then $n = 30$ and $5n = 150$ blocks.

D. The numbers grow by 2 in the ■s column, by 3 in the □s column, and by 5 in the Total column. These numbers are in front of the n in the generalizations $2n$, $3n$, and $5n$.

E. Yes; 110 is divisible by 5. The 22nd design will have 110 blocks.

F. 135
If $5n = 225$ blocks, then $n = 45$ and $3n = 135$ □s.

G. When you add $3n$ to $2n$, you have $5n$; i.e., $2n + 3n = 5n$.

Design	■s	□s	Total
1	2	3	5
2	4	6	10
3	6	9	15
4	8	12	20
5	10	15	25
⋮	⋮	⋮	⋮
10	20	30	50
⋮	⋮	⋮	⋮
20	40	60	100
⋮	⋮	⋮	⋮
n	2n	3n	5n

Pattern 22 Answers

A. 25, 26, 51
If $n = 25$ ■s, then $n + 1 = 26$ □s, and $2n + 1 = 51$.

B. The ■s are always in one row with n blocks in each row for a total of n ■s. The □s are also always in one row with $n + 1$ blocks in each row for a total of $n + 1$ □s.

C. 85
If $n = 42$, then $2n + 1 = 85$ blocks.

D. No; 86 is an even number. Our total number is always odd.

E. 52, 53
Since $2n + 1 = 105$, $n = 52$ and $n + 1 = 53$ □s.

F. 401
If $n = 200$, then $2n + 1 = 401$.

G. When we add the generalization for the ■s, n, with the generalization for the □s, $n + 1$, we have two ns or $2n$ plus 1: $n + (n + 1) = 2n + 1$.

Design	■s	□s	Total
1	1	2	3
2	2	3	5
3	3	4	7
4	4	5	9
5	5	6	11
⋮	⋮	⋮	⋮
10	10	11	21
⋮	⋮	⋮	⋮
20	20	21	41
⋮	⋮	⋮	⋮
n	n	n + 1	2n + 1

Pattern 23 Answers

A. 36, 38, 74
If $n = 18$, then $2n = 36$ ■s, $2n + 2 = 38$ □s, and $4n + 2 = 74$ blocks.

B. The ■s always have 2 blocks in a column and there are n columns for a total of $2n$ ■s. The □s also always have 2 blocks in a column, and there are $n + 1$ columns for a total of $2(n + 1)$ or $2n + 2$ □s.

C. 122
If $2n + 2 = 62$ □s, then $n = 30$ and $4n + 2 = 122$ blocks.

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D. 4. The 4 is the number in front of the n in our generalization of $4n + 2$.

E. 82
If $4n + 2 = 162$ blocks, then $n = 40$ and $2n + 2 = 82$ □s.

F. 174
If $2n + 2 = 88$ □s, then $n = 43$ and $4n + 2 = 174$ blocks.

G. When you add $2n$ and $2n + 2$, you have four n's, or $4n$, plus the number 2. Therefore, $2n + (2n + 2) = 4n + 2$.

Design	■s	□s	Total
1	2	4	6
2	4	6	10
3	6	8	14
4	8	10	18
5	10	12	22
⋮	⋮	⋮	⋮
10	20	22	42
⋮	⋮	⋮	⋮
20	40	42	82
⋮	⋮	⋮	⋮
n	2n	2(n + 1) or 2n + 2	2(2n + 1) or 4n + 2

Pattern 24 Answers

A. 28, 113, 141
Since $n = 28$ ■s, $4n + 1 = 113$ □s, and $5n + 1 = 141$ blocks.

B. The ■s are the counting numbers. The □s are formed by adding 4 to each number, starting with 5. The totals are formed by adding 5 to each number, starting with 6.

C. No, we will never use 91 □s; there is no whole number n that satisfies $4n + 1 = 91$. We will use exactly 121 □s in the 30th design because $4 \times 30 + 1 = 121$.

D. The numbers grow by 1, 4, and 5. These are the numbers in front of n for the generalizations n, $4n + 1$, and $5n + 1$.

E. 50, 201
If $5n + 1 = 251$ blocks, $n = 50$ ■s and $4n + 1 = 201$ □s.

F. 47
If $4n + 1 = 189$ □s, $n = 47$ ■s.

G. When you add n and $4n + 1$, you have five n's, or $5n$, and the number 1 for a result of $n + (4n + 1) = 5n + 1$, our generalization.

Design	■s	□s	Total
1	1	5	6
2	2	9	11
3	3	13	16
4	4	17	21
5	5	21	26
⋮	⋮	⋮	⋮
10	10	41	51
⋮	⋮	⋮	⋮
20	20	81	101
⋮	⋮	⋮	⋮
n	n	4n + 1	5n + 1

Pattern 25 Answers

A. 24, 46, 70
If $n = 23$, then $n + 1 = 24$ ■s, $2n = 46$ □s, and $3n + 1 = 70$ blocks.

B. The ■s are the counting numbers beginning with 2. The □s are the even numbers, or multiples of 2, beginning with 2. The total numbers are found by adding 3 to each number, starting with 4.

C. The □s are always in 2 columns with n □s in each column for a total of $2n$ □s.

D. The totals grow by 3. The 3 is the number in front of the n in our generalization of $3n + 1$.

E. 200
If $3n + 1 = 301$ blocks, then $n = 100$ and $2n = 200$ □s.

F. No. There is no whole number n that satisfies $3n + 1 = 401$. Dividing 400 by 3 does not result in a whole number.

Design	■s	□s	Total
1	2	2	4
2	3	4	7
3	4	6	10
4	5	8	13
5	6	10	16
⋮	⋮	⋮	⋮
10	11	20	31
⋮	⋮	⋮	⋮
20	21	40	61
⋮	⋮	⋮	⋮
n	n + 1	2n	3n + 1

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Pattern 26 Answers

Design	■s	□s	Total
1	2	6	8
2	3	7	10
3	4	8	12
4	5	9	14
5	6	10	16
⋮	⋮	⋮	⋮
10	11	15	26
⋮	⋮	⋮	⋮
20	21	25	46
⋮	⋮	⋮	⋮
n	n + 1	n + 5	2(n + 3) or 2n + 6

A. 35, 39, 74
If $n = 34$, then $n + 1 = 35$ ■s, $n + 5 = 39$ □s, and $2n + 6 = 74$ blocks.

B. The ■s and the □s columns begin with 2 and 6, respectively, and increase by 1. This results in a pattern made of the counting numbers. The totals column increases by 2s; the pattern is made up of the multiples of 2, or even numbers, beginning with the number 8.

C. 144
If $n + 1 = 70$ ■s, then $n = 69$ and $2n + 6 = 144$ blocks.

D. No. The total is always an even number. There is no whole number n that satisfies $2n + 6 = 189$.

E. 166
If $n + 5 = 85$ □s, then $n = 80$ and $2n + 6 = 166$ blocks.

F. 130, 134
If $2n + 6 = 264$ blocks, then $n = 129$, $n + 1 = 130$ ■s, and $n + 5 = 134$ □s.

G. 351, 355, 706
If $n = 350$, then $n + 1 = 351$ ■s, $n + 5 = 355$ □s, and $2n + 6 = 706$ blocks.

H. When you add $n + 1$ and $n + 5$, you have two n's, or $2n$, and $5 + 1$, or 6. Therefore, $(n + 1) + (n + 5) = 2n + 6$.

Pattern 27 Answers

Design	■s	□s	Total
1	1	3	4
2	2	5	7
3	3	7	10
4	4	9	13
5	5	11	16
⋮	⋮	⋮	⋮
10	10	21	31
⋮	⋮	⋮	⋮
20	20	41	61
⋮	⋮	⋮	⋮
n	n	2n + 1	3n + 1

A. 23, 47, 70
If $n = 23$ ■s, then $2n + 1 = 47$ □s, and $3n + 1 = 70$ blocks.

B. The □s are always in two rows. There are always n □s in the top row and $n + 1$ □s in the bottom row, so we have $n + (n + 1)$ or $2n + 1$ for the generalization for the number of □s.

C. The column numbers grow by 1, 2, and 3. These are the numbers before n in the generalizations n, $2n + 1$, and $3n + 1$.

D. 130
If $n = 43$ ■s, then $3n + 1 = 130$ □s.

E. No. There is no whole number n that satisfies $3n + 1 = 150$.

F. 80, 161
If $3n + 1 = 241$ blocks, then $3n = 240$. Therefore $n = 80$ ■s, and $2n + 1 = 161$ blocks.

G. When you add n and $2n + 1$, you have three n's, or $3n$, and the number 1, so $n + (2n + 1) = 3n + 1$.

Pattern 28 Answers

A. 26, 76, 102
If $n = 25$, then $n + 1 = 26$ ■s, $3n + 1 = 76$ □s, and $4n + 2 = 102$ blocks.

B. There are always 2 columns with only □s. There are n □s in each of these two columns for a subtotal of $2n$ □s. There are also $n + 1$ □s in a row along the top. The total will therefore be $2n + (n + 1)$ or $3n + 1$ □s.

C. 154
If $n + 1 = 39$ ■s, then $n = 38$ and $4n + 2 = 154$ blocks.

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D. The numbers grow by 1, 3, and 4 in each column. These are the numbers in front of n in the generalizations $n + 1$, $3n + 1$, and $4n + 2$.

E. No. There is no whole number n that satisfies either $4n + 2 = 199$ or $4n + 2 = 200$, though 202 will work.

F. 66
If $4n + 2 = 262$ blocks, then $n = 65$ and $n + 1 = 66$ ■s.

G. When you add $n + 1$ and $3n + 1$ you have four n's, or $4n$, and $1 + 1$, or 2, so $(n + 1) + (3n + 1) = 4n + 2$.

Design	■s	□s	Total
1	2	4	6
2	3	7	10
3	4	10	14
4	5	13	18
5	6	16	22
⋮	⋮	⋮	⋮
10	11	31	42
⋮	⋮	⋮	⋮
20	21	61	82
⋮	⋮	⋮	⋮
n	n + 1	3n + 1	4n + 2

Pattern 29 Answers

A. 48
If $2n = 46$ ■s, then $2n + 2 = 48$ □s.

B. Disregarding the top two blocks, you can push the blocks toward the center to form a rectangle of base 4, height n. The total number of blocks equals the dimensions of the rectangle plus the top 2 blocks: $4n + 2$

C. 402
If $2n = 200$ ■s, then $n = 100$ and $4n + 2 = 402$ blocks.

D. 95 blocks: No; there is no whole number n that satisfies the equation $4n + 2 = 95$.
96 blocks: No; there is no whole number n that satisfies $4n + 2 = 96$.

E. 62
If $4n + 2 = 122$ blocks, then $n = 30$ and $2n + 2 = 62$ □s.

F. The numbers grow by 2s in the ■s and □s columns and by 4s in the total column. These are the numbers in front of n in the generalizations $2n$, $2n + 2$, and $4n + 2$.

G. 174, 176
If $n = 87$, then $2n = 174$ ■s and $2n + 2 = 176$ □s.

H. When you add $2n$ and $2n + 2$ you have four n's, or $4n$, and the number 2. Therefore, $2n + (2n + 2) = 4n + 2$.

Design	■s	□s	Total
1	2	4	6
2	4	6	10
3	6	8	14
4	8	10	18
5	10	12	22
⋮	⋮	⋮	⋮
10	20	22	42
⋮	⋮	⋮	⋮
20	40	42	82
⋮	⋮	⋮	⋮
n	2n	2n + 2	4n + 2

Pattern 30 Answers

A. 30, 66, 96
If $n = 30$ ■s, then $2n + 6 = 66$ □s, and $3n + 6 = 96$ blocks.

B. The top and bottom rows always have $n + 2$ □s each for a subtotal of $2(n + 2)$ or $2n + 4$. When you add the constant 2 □s at the ends of the middle row, you get a total of $(2n + 6)$ □s. The total number of blocks always forms a rectangle with dimensions $3 \times (n + 2)$ or $3n + 6$.

C. No. We always use an even number of □s.

D. 98
If $n = 46$ ■s, then $2n + 6 = 98$ □s.

E. No. There is no whole number n that satisfies $3n + 6 = 125$. We will use 126 blocks, but not 125.

F. 156

Design	■s	□s	Total
1	1	8	9
2	2	10	12
3	3	12	15
4	4	14	18
5	5	16	21
⋮	⋮	⋮	⋮
10	10	26	36
⋮	⋮	⋮	⋮
20	20	46	66
⋮	⋮	⋮	⋮
n	n	2n + 6	3n + 6

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If $2n + 6 = 106$ □s, then $n = 50$ and $3n + 6 = 156$ blocks.

G. 11×3
14×3
22×3
$(n + 2) \times 3$

Pattern 31 Answers

Design	■s	□s	Total
1	3	5	8
2	6	7	13
3	9	9	18
4	12	11	23
5	15	13	28
⋮	⋮	⋮	⋮
10	30	23	53
⋮	⋮	⋮	⋮
20	60	43	103
⋮	⋮	⋮	⋮
n	3n	2n + 3	5n + 3

A. 90, 63, 153
If $n = 30$, then $3n = 90$ ■s, $2n + 3 = 63$ □s, and $5n + 3 = 153$ blocks.

B. The ■s always have 3 "arms," (right, left, and middle) with n ■s in each arm for a total of $3n$ ■s. The □s have two arms, each with $n + 1$ □s plus 1 □ on top for a total of $2(n + 1) + 1 = 2n + 2 + 1$, or $2n + 3$ □s. The total has 5 arms, each with n blocks, plus 3 extra □s on top for a total of $5n + 3$ blocks.

C. 163
If $3n = 96$ ■s, then $n = 32$ and $5n + 3 = 163$ blocks.

D. The numbers increase by 3, 2, and 5. These are the numbers in front of the n in the generalizations $3n$, $2n + 3$, and $5n + 3$.

E. 75
If $5n + 3 = 128$ blocks, then $n = 25$ and $3n = 75$ ■s.

F. 150
If $2n + 3 = 103$ □s, then $n = 50$ and $3n = 150$ ■s.

G. When you add $3n$ and $2n + 3$, you have five n's, or $5n$, and the number 3. Therefore, $3n + (2n + 3) = 5n + 3$.

Pattern 32 Answers

Design	■s	□s	Total
1	2	10	12
2	4	12	16
3	6	14	20
4	8	16	24
5	10	18	28
⋮	⋮	⋮	⋮
10	20	28	48
⋮	⋮	⋮	⋮
20	40	48	88
⋮	⋮	⋮	⋮
n	2n	2n + 8	4n + 8

A. 58, 66, 124
If $n = 29$, then $2n = 58$ ■s, $2n + 8 = 66$ □s, and $4n + 8 = 124$.

B. The ■s are always in a rectangle with dimensions $2 \times n$ or $2n$. The □s are in two columns of $n + 2$ blocks each or $2(n + 2)$ plus the two constant □s in each of the top and bottom rows; they total $2n + 8$. The overall arrangement of blocks is always a $4 \times (n + 2)$ rectangle, which gives a total of $4n + 8$ blocks.

C. 74
If $2n = 66$ ■s, then $2n + 8 = 74$ □s.

D. 255 blocks: No; the total is always an even number.
256 blocks: Yes (for the 62nd design); $(4 \times 62) + 8 = 256$.

E. The numbers increase by 2, 2, and 4. These are the numbers in front of n in the generalizations $2n$, $2n + 8$, and $4n + 8$.

F. 76, 84
If $4n + 8 = 160$ blocks, then $n = 38$, $2n = 76$ ■s, and $2n + 8 = 84$ □s.

G. 4×12
4×22
$4 \times (n + 2)$

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Pattern 33 Answers

Design	■s	□s	Total
1	1	4	5
2	3	8	11
3	5	12	17
4	7	16	23
5	9	20	29
⋮	⋮	⋮	⋮
10	19	40	59
⋮	⋮	⋮	⋮
20	39	80	119
⋮	⋮	⋮	⋮
n	2n – 1	4n	6n – 1

A. 45, 92, 137
If $n = 23$, then $2n - 1 = 45$ ■s, $4n = 92$ □s, and $6n - 1 = 137$ blocks.

B. The ■s consist of two arms, each with $n - 1$ ■s plus the one ■ in the center for a total of $2(n - 1) + 1 = (2n - 2) + 1 = 2n - 1$ ■s. We could also think of each arm as having n ■s, except that one ■ is missing, so there are $2n - 1$ ■s. The □s consist of 4 arms: if you move the bottom □ up, each arm can have n □s for a total of 4n □s. The entire block arrangement can be thought of as having 6 arms of n blocks each, except one arm is missing a block. The total is therefore $6n - 1$ blocks.

C. 99
If $4n = 200$ □s, then $n = 50$, and $2n - 1 = 99$ ■s.

D. The numbers in the columns grow by 2, 4, and 6. These are the numbers in front of the n in the generalizations $2n - 1$, $4n$, and $6n - 1$.

E. When you add an even number and an odd number, the answer is always an odd number: $2n + (2n - 1) = 4n - 1$.

F. 93
If $6n - 1 = 281$ blocks, then $n = 47$ and $2n - 1 = 93$ ■s.

G. 280
If $2n - 1 = 139$ ■s, then $n = 70$ and $4n = 280$ □s.

Pattern 34 Answers

Design	■s	□s	Total
1	1	8	9
2	3	12	15
3	5	16	21
4	7	20	27
5	9	24	33
⋮	⋮	⋮	⋮
10	19	44	63
⋮	⋮	⋮	⋮
20	39	84	123
⋮	⋮	⋮	⋮
n	2n – 1	4n + 4	6n + 3

A. 65, 136, 201
If $n = 33$, then $2n - 1 = 65$ ■s, $4n + 4 = 136$ □s, and $6n + 3 = 201$ blocks.

B. The ■s are always in a row of $2n - 1$ blocks. The □s are in two rows, each with $2n + 1$ blocks resulting in $4n + 2$ plus the constant 2 □s in the middle row for a total of $4n + 4$ □s. The total arrangement of blocks is always a rectangle with dimensions of $3 \times (2n + 1)$ or a total of $6n + 3$.

C. No, we never use odd numbers of □s (multiples of 4 are always even numbers). No, the total number will never be even; $6n$ is always an even number, so when you add 3, you will have an odd number.

D. 120
If $2n - 1 = 57$ ■s, then $n = 29$ and $4n + 4 = 120$ □s.

E. 73, 152
If $6n + 3 = 225$ blocks, then $n = 37$; therefore, $2n - 1 = 73$ ■s and $4n + 4 = 152$ □s.

F. 19×3
21×3
41×3
$(2n + 1) \times 3$

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Pattern 35 Answers

Design	■s	□s	Total
1	3	12	15
2	6	14	20
3	9	16	25
4	12	18	30
5	15	20	35
⋮	⋮	⋮	⋮
10	30	30	60
⋮	⋮	⋮	⋮
20	60	50	110
⋮	⋮	⋮	⋮
n	3n	2n + 10	5(n + 2) or 5n + 10

A. 81, 64, 145
If $n = 27$, then $3n = 81$ ■s, $2n + 10 = 64$ □s, and $5n + 10 = 145$ blocks.

B. The top and bottom rows have 5 □s each. There are n remaining rows each having two □s (at the ends) for $2n$ □s. The total number of □s is therefore $2n + 10$. The ■s are always in a rectangle with the dimensions of $3 \times n$ for a total of $3n$ ■s. The total block arrangement is a rectangle with the dimensions $5 \times (n + 2)$, or $5n + 10$ blocks.

C. Yes. We sometimes use odd numbers because every other multiple of 3 is an odd number. No, we will not use an odd number of □s; the numbers of □s are multiples of 2 and are, therefore, always even.

D. 72
If $3n = 93$ ■s, then $n = 31$ and $2n + 10 = 72$ □s.

E. 90
If $5n + 10 = 210$, then $n = 40$ and $2n + 10 = 90$ □s.

F. 5×10
5×12
5×22
$5 \times (n + 2)$

G. When you add $3n$ to $2n + 10$, you have five n's or $5n$. You also have the number 10 in the sum, so $3n + (2n + 10) = 5n + 10$.

Pattern 36 Answers

Design	■s	□s	Total
1	4	5	9
2	8	17	25
3	12	29	41
4	16	41	57
5	20	53	73
⋮	⋮	⋮	⋮
10	40	113	153
⋮	⋮	⋮	⋮
20	80	233	313
⋮	⋮	⋮	⋮
n	4n	12n – 7	16n – 7

A. 120, 353, 473
If $n = 30$, then $4n = 120$ ■s, $12n - 7 = 353$ □s, and $16n - 7 = 473$ blocks.

B. The ■s are always in four arms, with n blocks each, for a total of $4n$ ■s. The □s are also in four arms, each with $3n - 2$ □s, for a subtotal of $4 \times (3n - 2)$ or $12n - 8$. Add the one block in the middle for a total of $12n - 7$ □s. The total number of blocks also lies in 4 arms, each with $4n - 2$, or $4 \times (4n - 2) = 16n - 8$. Add the one block in the middle for a grand total of $16n - 7$ blocks.

C. 161
If $4n = 56$ ■s, then $n = 14$ and $12n - 7 = 161$ □s.

D. No; no whole number n satisfies $4n = 90$, i.e., 90 is not a multiple of 4.

E. The numbers grow by 4, 12, and 16. These are the numbers in front of the n in the generalizations $4n$, $12n - 7$, and $16n - 7$.

F. 160, 473, 649
If $16n - 7 = 633$ blocks, then $n = 40$. Therefore, $4n = 160$ ■s, and $12n - 7 = 473$ □s. Since the totals grow by 16, the next total will be $633 + 16 = 649$ blocks.

G. 100
If $12n - 7 = 293$ □s, then $n = 25$ and $4n = 100$ ■s.

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Pattern 37 Answers

Design	■s	□s	Total
1	1	0	1
2	5	8	13
3	9	16	25
4	13	24	37
5	17	32	49
⋮	⋮	⋮	⋮
10	37	72	109
⋮	⋮	⋮	⋮
20	77	152	229
⋮	⋮	⋮	⋮
n	4n – 3	8n – 8	12n – 11

A. 97, 192, 289
If $n = 25$, then $4n - 3 = 97$ ■s, $8n - 8 = 192$ □s, and $12n - 11 = 289$ blocks.

B. The ■s are in 4 arms with $n - 1$ blocks each, for a subtotal of $4n - 4$ blocks. Add the one block in the middle for a total of $4n - 3$ ■s. The □s are in 4 arms with $2(n - 1)$ blocks each for a total of $8n - 8$ □s. The combined block arrangement has $3(n - 1)$ blocks per arm, or $12n - 12$, plus the one block in the middle for a total of $12n - 11$ blocks.

C. 232
If $4n - 3 = 117$ ■s, then $n = 30$ and $8n - 8 = 232$ □s.

D. The numbers grow by 4, 8, and 12. These are the numbers in front of the n in our generalizations $4n - 3$, $8n - 8$, and $12n - 11$.

E. 157, 312
If $12n - 11 = 469$ blocks, then $n = 40$, $4n - 3 = 157$ ■s, and $8n - 8 = 312$ □s.

F. ■s: No; for all values of n, $4n$ gives an even number, and subtracting 3 from $4n$ gives us an odd number.
Total: No; we subtract 11 from $12n$, which gives us an odd number.

G. No. There is no whole number n that satisfies $4n - 3 = 315$.

H. When you add the n's from the numbers of ■s and □s, we have $4n + 8n = 12n$. We also add –3 and –8 for –11, resulting in a total of $12n - 11$.

Pattern 38 Answers

Design	■s	□s	Total
1	5	4	9
2	10	10	20
3	15	20	35
4	20	34	54
5	25	52	77
⋮	⋮	⋮	⋮
10	50	202	252
⋮	⋮	⋮	⋮
20	100	802	902
⋮	⋮	⋮	⋮
n	5n	$2n^2 + 2$	$2n^2 + 5n + 2$

A. 150, 1802, 1952
If $n = 30$, then $5n = 150$ ■s, $2n^2 + 2 = 1802$ □s, and $2n^2 + 5n + 2 = 1952$ blocks.

B. In addition to the two □s at the top center, there are always two arrangements of □s, one on each lower side; each has the dimensions $n \times n$. Therefore, $2n^2$ plus the two top blocks gives us the generalization for □s: $2n^2 + 2$.

C. ■s: Yes; the numbers are multiples of 5 and some are odd.
□s: No; multiplying by 2 and then adding 2 results in an even number.

D. 452
If $5n = 75$ ■s, then $n = 15$ and $2n^2 + 2 = 452$ □s.

E. 95 ■s: Yes; since 19 satisfies the equation $5n = 95$, the 19th design will use 95 □s.
95 □s: No; the □s are always even numbers.
100 □s: Yes; since 7 satisfies $2n^2 + 2 = 100$, the 7th design will use 100 □s.
200 □s: No; there is no whole number n that satisfies $2n^2 + 2 = 200$.

F. 40
If $2n^2 + 2 = 130$ □s, then $n = 8$ and $5n = 40$ ■s.

G. 11×7
13×8
21×12
41×22
$(2n + 1) \times (n + 2)$

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When you multiply $(2n + 1) \times (n + 2)$, you get our total generalization, i.e., $(2n + 1) \times (n + 2) = 2n^2 + 5n + 2$.

Pattern 39 Answers

A. 1378, 1431
We know that $(n + 1)^2 = 2809$ blocks. Let students use calculators to find $n + 1 = 53$, by trial and error. Therefore, $n = 52$, $n(n+1)/2 = 1378$ ■s, and $(n+1)(n+2)/2 = 1431$ □s.

Design	■s	□s	Total
1	1	3	4
2	3	6	9
3	6	10	16
4	10	15	25
5	15	21	36
⋮	⋮	⋮	⋮
10	55	66	121
⋮	⋮	⋮	⋮
20	210	231	441
⋮	⋮	⋮	⋮
n	$\frac{n(n+1)}{2}$	$\frac{(n+1)(n+2)}{2}$	$(n+1)^2$

B. The numbers in the totals column are found by adding successive odd numbers: add 5, then 7, then 9, etc. The numbers in the totals column are what we call the square numbers.

C. The ■s are found by adding one more each time, i.e., first 2, then 3, then 4, etc. We call the numbers the triangle numbers; the ■s form triangles.

D. You can rotate the first n columns of blocks up and counterclockwise 90° to form the top right corner of a square.

E. 7×7
8×8
11×11
21×21
$(n + 1) \times (n + 1)$
The dimensions of each rearranged square is $(n + 1) \times (n + 1)$, which equals the total generalization of $(n + 1)^2$.

Pattern 40 Answers

A. 89, 1936, 2025
If $n = 23$, then $4n - 3 = 89$ ■s, $(2n - 2)^2 = 1936$ □s, and $(2n - 1)^2 = 2025$ blocks.

Design	■s	□s	Total
1	1	0	1
2	5	4	9
3	9	16	25
4	13	36	49
5	17	64	81
⋮	⋮	⋮	⋮
10	37	324	361
⋮	⋮	⋮	⋮
20	77	1444	1521
⋮	⋮	⋮	⋮
n	4n – 3	$4(n-1)^2$ or $(2n-2)^2$	$(2n-1)^2$

B. The numbers in the ■s column grow by 4. This 4 is the number in front of the n in our generalization of $4n - 3$.

C. If you eliminate the ■s, you can take the top $n - 2$ rows and the bottom $n - 2$ rows of □s and squeeze them together to fill in the rows immediately below and above them; each row will have $2n - 2$ □s. The dimensions of the new square will be $(2n - 2) \times (2n - 2)$, or $(2n - 2)^2$, which is the □ generalization.

D. 2304
If $4n - 3 = 97$ ■s, then $n = 25$ and $4 \times (n - 1)^2 = 2304$ □s.

E. 11×11
19×19
39×39
$(2n - 1) \times (2n - 1)$
The dimensions of the square equal the generalization for the total number of blocks: $(2n - 1) \times (2n - 1) = (2n - 1)^2$.

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SECTION 3

Pattern 41 Answers

Design	■s	□s	▩s	Total
1	1	2	2	5
2	2	3	4	9
3	3	4	6	13
4	4	5	8	17
5	5	6	10	21
⋮	⋮	⋮	⋮	⋮
10	10	11	20	41
⋮	⋮	⋮	⋮	⋮
20	20	21	40	81
⋮	⋮	⋮	⋮	⋮
n	n	n + 1	2n	4n + 1

A. 27, 28, 54, 109
If n = 27 ■s, $n + 1$ = 28 □s, $2n$ = 54 ▩s, and $4n + 1$ = 109 blocks.

B. The ▩s are in n columns with 2 ▩s per column for a total of $2n$ ▩s. For the total, if you move the □s from the bottom row to the tops of their neighboring columns, you will have n columns with 4 blocks each, or $4n$, plus one □ left over for an overall total of $4n + 1$ blocks.

C. 30, 31
If $2n$ = 60 ▩s, then n = 30 ■s and $n + 1$ = 31 □s.

D. The numbers grow by 2 in the ▩s column and by 4 in the totals column. These are the numbers in front of the n in the generalizations $2n$ and $4n + 1$.

E. 70, 71, 140
If $4n + 1$ = 281 blocks, then n = 70 ■s, $n + 1$ = 71 □s, and $2n$ = 140 ▩s.

F. When you add n and $n + 1$ and $2n$, you have four n's, or $4n$, plus the number 1. Therefore, $n + (n + 1) + 2n = 4n + 1$.

Pattern 42 Answers

Design	■s	□s	▩s	Total
1	1	2	3	6
2	2	3	6	11
3	3	4	9	16
4	4	5	12	21
5	5	6	15	26
⋮	⋮	⋮	⋮	⋮
10	10	11	30	51
⋮	⋮	⋮	⋮	⋮
20	20	21	60	101
⋮	⋮	⋮	⋮	⋮
n	n	n + 1	3n	5n + 1

A. 35, 36, 105, 176
If n = 35 ■s, then $n + 1$ = 36 □s, $3n$ = 105 ▩s, and $5n + 1$ = 176 blocks.

B. The ▩s are always in clusters of 3, and there are always n clusters for a total of $3n$. (Also, the ▩s are always in two rows, one with n, the other with $2n$; $n + 2n = 3n$ ▩s.) The combined blocks are in n clusters of 5, with one white block left over for a total of $5n + 1$ blocks. You can also see the arrangement as three rows of blocks, one with n, one with $2n$, and one with $2n + 1$, for a total of $5n + 1$ blocks.

C. 225, 376
If n = 75, then $3n$ = 225 ▩s and $5n + 1$ = 376.

D. The numbers grow by 3 in the ▩s column and by 5 in the totals column. These are the numbers in front of the n in the generalizations $3n$ and $5n + 1$.

E. 57, 58, 171
If $5n + 1$ = 286 blocks, then n = 57 ■s, $n + 1$ = 58 □s, and $3n$ = 171 blocks altogether.

F. 103 ▩s: No; 103 is not a multiple of 3.
103 □s: Yes; we will use 103 □s in the 102nd design.
340 blocks: No; there is no whole number n that satisfies $5n + 1 = 340$.
341 blocks: Yes; the 68th design will use 341 blocks.
446 blocks: Yes; the 89th design will use 446 blocks.

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Pattern 43 Answers

Design	■s	□s	▩s	Total
1	2	6	4	12
2	4	8	6	18
3	6	10	8	24
4	8	12	10	30
5	10	14	12	36
⋮	⋮	⋮	⋮	⋮
10	20	24	22	66
⋮	⋮	⋮	⋮	⋮
20	40	44	42	126
⋮	⋮	⋮	⋮	⋮
n	2n	2n + 4	2n + 2	6n + 6

A. 56, 60, 58, 174
If $n = 28$, then $2n = 56$ ■s, $2n + 4 = 60$ □s, $2n + 2 = 58$ ▩s, and $6n + 6 = 174$ blocks.

B. The □s include two diagonals, with $n + 1$ blocks each, and the constant 2 blocks not on either diagonal, giving a total of $2n + 4$ □s. The ▩s also include two diagonals, with n blocks each, and the constant 2 blocks not on either diagonal, giving a total of $2n + 2$ ▩s. The total number of blocks includes six diagonals, with n blocks on each, plus an extra rectangle of 6 blocks. The total is $6n + 6$.

C. 84, 82
If $2n = 80$ ■s, then $2n + 4 = 84$ □s and $2n + 2 = 82$ ▩s.

D. The column numbers grow by 2, 2, 2, and 6. These are the numbers in front of the n in the generalizations $2n$, $2n + 4$, $2n + 2$, and $6n + 6$.

E. 160, 164, 162
If $6n + 6 = 486$ blocks, then $n = 80$, $2n = 160$ ■s, $2n + 4 = 164$ □s, and $2n + 2 = 162$ ▩s.

F. The numbers in the Total column can each be divided by 6 to get a whole number; the numbers are called the multiples of 6.

G. When we add $2n$, $2n + 4$, and $2n + 2$ we have six n's, or $6n$, plus the numbers 4 and 2 for $4 + 2 = 6$. Therefore, the total is $2n + (2n + 4) + (2n + 2) = 6n + 6$.

Pattern 44 Answers

Design	■s	□s	▩s	Total
1	1	3	6	10
2	2	6	8	16
3	3	9	10	22
4	4	12	12	28
5	5	15	14	34
⋮	⋮	⋮	⋮	⋮
10	10	30	24	64
⋮	⋮	⋮	⋮	⋮
20	20	60	44	124
⋮	⋮	⋮	⋮	⋮
n	n	3n	2n + 4	6n + 4

A. 27, 81, 58, 166
Since $n = 27$ ■s, $3n = 81$ □s, $2n + 4 = 58$ ▩s, and $6n + 4 = 166$ blocks.

B. The □s are always arranged in a T with 3 arms of n blocks each for a total of $3n$ □s. The ▩s are always in two clusters of $n + 2$ blocks each for a total of $2n + 4$ ▩s. The total number of blocks is arranged in three parts: one center column and two side arms (counting the horizontal □s and n of the vertical ▩s). Each of those three parts has $2n$ blocks for a subtotal of $3 \times 2n = 6n$, but there are always 4 gray blocks left over. The grand total is $6n + 4$ blocks.

C. 201, 138
If $n = 67$ ■s, then $3n = 201$ □s and $2n + 4 = 138$ ▩s.

D. The numbers grow by 1, 3, 2, and 6. Those are the numbers in front of the n in our final answers: n, $3n$, $2n + 4$, and $6n + 4$.

E. 55, 165, 114
If $6n + 4 = 334$ blocks, then $n = 55$ ■s, $3n = 165$ □s, and $2n + 4 = 114$ ▩s.

F. 155: No; the total is always an even number.
160: Yes; the 26th design will use 160 blocks.

G. When we add n, $3n$, and $2n + 4$, we have six n's, or $6n$, plus the 4 for a total of $6n + 4$ blocks.

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Pattern 45 Answers

Design	■s	□s	▪s	Total
1	3	8	7	18
2	5	12	11	28
3	7	16	15	38
4	9	20	19	48
5	11	24	23	58
⋮	⋮	⋮	⋮	⋮
10	21	44	43	108
⋮	⋮	⋮	⋮	⋮
20	41	84	83	208
⋮	⋮	⋮	⋮	⋮
n	2n + 1	4n + 4	4n + 3	10n + 8

A. 65, 132, 131, 328
If $n = 32$, then $2n + 1 = 65$ ■s, $4n + 4 = 132$ □s, $4n + 3 = 131$ ▪s, and $10n + 8 = 328$ blocks.

B. The ■s are always in two branches with n blocks on each side and one block in the middle for a total of $2n + 1$ ■s. The □s are in four branches of n with an additional four constant blocks in a plus sign (+) shape at top. This results in a total of $4n + 4$ □s. The ▪s are in four branches: the top two branches each have $n + 1$ blocks, and the lower two branches each have $n - 1$ blocks; there is an extra ▪ at the apex and another at each lower corner. This gives $2(n + 1) + 2(n - 1) + 3 = 2n + 2 + 2n - 2 + 3 = 4n + 3$. The entire set of blocks is arranged as five pairs of branches. Starting with the lowest pair we have $2 \times (n - 1) + 1$, then $(2 \times n) + 1$, then $2 \times (n + 1) + 1$, then $2 \times (n + 1) + 1$, then $2 \times (n + 1) = 2n - 2 + 1 + 2n + 1 + 2n + 2 + 1 + 2n + 2 + 1 + 2n + 2 = 10n + 8$

C. 164, 163
Since $2n + 1 = 81$ ■s, $n = 40$, $4n + 4 = 164$ □s, and $4n + 3 = 163$ ▪s.

D. The column numbers grow by 2, 4, 4, and 10. These are the numbers in front of the n in the generalizations $2n + 1$, $4n + 4$, $4n + 3$, and $10n + 8$.

E. 101, 204, 203
If $10n + 8 = 508$ blocks, then $n = 50$, $2n + 1 = 101$ ■s, $4n + 4 = 204$ □s, and $4n + 3 = 203$ ▪s.

F. 80 ■s: No; the ■s are always an odd number.
80 □s: Yes; the 19th design will use 80 □s.
80 ▪s: No; the ▪s are always an odd number.
80 blocks: No; there is no whole number n that satisfies $10n + 8 = 80$ blocks.
180 blocks: No; there is no whole number n that satisfies $10n + 8 = 180$ blocks.

G. When we add $2n + 1$, $4n + 4$, and $4n + 3$, we have ten n's, or $10n$, plus $1 + 4 + 3 = 8$, or $10n + 8$.

Pattern 46 Answers

Design	■s	□s	▪s	Total
1	3	4	6	13
2	4	8	10	22
3	5	12	14	31
4	6	16	18	40
5	7	20	22	49
⋮	⋮	⋮	⋮	⋮
10	12	40	42	94
⋮	⋮	⋮	⋮	⋮
20	22	80	82	184
⋮	⋮	⋮	⋮	⋮
n	n + 2	4n	4n + 2	9n + 4

A. 37, 140, 142, 319
If $n = 35$, then $n + 2 = 37$ ■s, $4n = 140$ □s, $4n + 2 = 142$ ▪s, and $9n + 4 = 319$ blocks.

B. The □s are always in two rectangles, each with dimensions of $2 \times n$, or in n columns of 4 each, for a total of $4n$ □s. The ▪s are in four clusters of n each plus 2 left over for a total of $4n + 2$ ▪s. The total number of blocks lies in 9 clusters of n with 4 blocks left over (2 ■s and 2 ▪s) for a grand total of $9n + 4$ blocks.

C. 300, 302
If $n + 2 = 77$ ▪s, then $n = 75$, $4n = 300$ □s, and $4n + 2 = 302$ ▪s.

D. The column numbers grow by 1, 2, 4, and 9. These are the numbers in front of the n in the generalizations $n + 2$, $4n$, $4n + 2$, and $9n + 4$.

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E. 52, 200, 202
If $9n + 4 = 454$ blocks, then $n = 50$, $n + 2 = 52$ ■s, $4n = 200$ □s, and $4n + 2 = 202$ ■s.

F. 91 □s: No; the □s are always an even number.
112 ■s: No; there is no whole number n that satisfies $4n + 2 = 112$.
112 blocks: Yes; the 12th design uses 112 blocks.

G. When we add $n + 2$, $4n$, and $4n + 2$, we have nine n's or $9n$ plus $2 + 2 = 4$ for a total of $9n + 4$.

Pattern 47 Answers

Design	■s	□s	■s	Total
1	2	3	4	9
2	4	5	7	16
3	6	7	10	23
4	8	9	13	30
5	10	11	16	37
⋮	⋮	⋮	⋮	⋮
10	20	21	31	72
⋮	⋮	⋮	⋮	⋮
20	40	41	61	142
⋮	⋮	⋮	⋮	⋮
n	2n	2n + 1	3n + 1	7n + 2

A. 76, 77, 115, 268
If $n = 38$, then $2n = 76$ ■s, $2n + 1 = 77$ □s, $3n + 1 = 115$ ■s, and $7n + 2 = 268$ blocks.

B. The ■s are always in a V shape with $n - 1$ blocks on each arm, or $2(n - 1)$, and a constant two ■s in the middle. Adding the two extra blocks, we get a total of $2(n - 1) + 2 = (2n - 2) + 2 = 2n$ ■s. The □s are also in a V shape with n blocks on each side and one extra at the bottom for a total of $2n + 1$ □s. The ■s are in two arms and one center column: arms have $n + 1$ ■s and the column has $n - 1$ ■s for a total of $(n + 1) + (n + 1) + (n - 1) = 3n + 1$.
If we ignore the blocks in the center column for now, the entire block design can be seen as having two lower arms each containing n blocks, two middle arms each with $n + 1$ blocks, and two upper arms each with $n - 1$ blocks. Add the blocks of the vertical column $n + 2$ and we get the following:
$2n + 2(n + 1) + 2(n - 1) + (n + 2) =$
$2n + 2n + 2 + 2n - 2 + n + 2 = 7n + 2$

C. 97, 145
If $2n = 96$ ■s, then $n = 48$, $2n + 1 = 97$ □s, and $3n + 1 = 145$ ■s.

D. The numbers grow by 2, 2, 3, and 7. These are the numbers in front of the n in the generalizations $2n$, $2n + 1$, $3n + 1$, and $7n + 2$.

E. 120, 121, 181
If $7n + 2 = 422$ blocks, then $n = 60$; therefore, $2n = 120$ ■s, $2n + 1 = 121$ □s, and $3n + 1 = 181$ ■s.

F. 115 ■s: No; the ■s are always even numbers.
115 □s: Yes; the 57th design will use 115 □s.
115 ■s: Yes; the 38th design will use 115 ■s.
310 total: Yes; the 44th design will use 310 blocks.

G. When we add $2n$, $2n + 1$, and $3n + 1$, we have $7n$'s plus $1 + 1 = 2$ for a total of $7n + 2$.

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Pattern 48 Answers

Design	■s	□s	▩s	Total
1	6	4	2	12
2	10	12	8	30
3	14	24	18	56
4	18	40	32	90
5	22	60	50	132
⋮	⋮	⋮	⋮	⋮
10	42	220	200	462
⋮	⋮	⋮	⋮	⋮
20	82	840	800	1722
⋮	⋮	⋮	⋮	⋮
n	$4n + 2$	$2n^2 + 2n$	$2n^2$	$4n^2 + 6n + 2$

A. 138, 2380, 2312, 4830
If $n = 34$, then $4n + 2 = 138$ ■s. Let the students use calculators to explore for the answers: $2n^2 + 2n = 2380$ □s; $2n^2 = 2312$ ▩s; $4n^2 + 6n + 2 = 4830$ blocks.

B. The ■s include four diagonals, with n blocks on each, plus the two constant blocks in the middle. This gives a total of $4n + 2$ ■s. The ▩s can be rearranged into two squares, one on each side of the design, each with dimensions of $n \times n$. The total number of ▩s is therefore $2n^2$.

C. 1300, 1250
If $4n + 2 = 102$ ■s, then $n = 25$, $2n^2 + 2n = 1300$ □s, and $2n^2 = 1250$ ▩s.

D. 124 ■s: No; there is no whole number n that satisfies $4n + 2 = 124$.
288 □s: No; there is no whole number n that satisfies $2n^2 + 2n = 288$.
288 ▩s: Yes; the 12th design uses 288 ▩s.
870 blocks: Yes; the 14th design uses 870 blocks.
Let the students use calculators; discuss solving $4n^2 + 6n + 2 = 870$ by trial and error by substituting 10, 11, 12, etc. If appropriate, discuss factoring: $2n^2 + 3n - 434 = 0$, $(2n + 31)(n - 14) = 0$. Again, students may use calculators.

E. 12×11
22×21
42×41
$(2n + 2) \times (2n + 1)$

F. The dimensions for the nth design are $(2n + 2) \times (2n + 1)$. This, when multiplied out, equals $4n^2 + 6n + 2$, which is the generalization for the total number of blocks. You may want to show one way the answer can be derived: $(2n + 2) \times (2n + 1) = (2n \times 2n) + (2n \times 1) + (2 \times 2n) + (2 \times 1) = 4n^2 + 2n + 4n + 2 = 4n^2 + 6n + 2$

Pattern 49 Answers

Design	■s	□s	▩s	Total
1	1	2	1	4
2	4	6	6	16
3	9	12	15	36
4	16	20	28	64
5	25	30	45	100
⋮	⋮	⋮	⋮	⋮
10	100	110	190	400
⋮	⋮	⋮	⋮	⋮
20	400	420	780	1600
⋮	⋮	⋮	⋮	⋮
n	n^2	$n(n + 1)$	$n(2n - 1)$	$(2n)^2$ or $4n^2$

A. 1225, 1260, 2415, 4900
If $n = 35$, then $n^2 = 1225$ ■s, $n(n + 1) = 1260$ □s, $n(2n - 1) = 2415$ ▩s, and $4n^2 = 4900$ blocks.

B. The ■s can always be rearranged into a square with dimensions $n \times n$, thus the total of n^2. The □s can be combined to form a rectangle with dimensions $(n + 1) \times n$, which is the □s generalization.

C. 342, 630, 1296
If $n^2 = 324$ ■s, then $n = 18$; $n^2 + n = 342$ □s, $n(2n - 1) = 630$ ▩s, and $(2n)^2 = 1296$ blocks.

D. 9×5
19×10
39×20
$(2n - 1) \times n$
The dimensions of the nth rectangle are $(2n - 1) \times n$. They are the same as the generalization for the number of ▩s: $2n^2 - n$.

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E. By moving the first $2n - 1$ columns up and counterclockwise 90°, you can make a square whose dimensions will be $2n \times 2n$. This is the same as the final generalization for the total number of blocks: $2n \times 2n = (2n)^2 = 4n^2$.

Pattern 50 Answers

A. 703, 741, 1482, 2926

If $n = 37$, then $\frac{n(n+1)}{2} = 703$ ■s, $\frac{(n+1)(n+2)}{2} = 741$ □s, $(n + 1)(n + 2) =$ 1482 ■s, and $2n^2 + 5n + 3 =$ 2926 blocks.

Design	■s	□s	■s	Total
1	1	3	6	10
2	3	6	12	21
3	6	10	20	36
4	10	15	30	55
5	15	21	42	78
⋮	⋮	⋮	⋮	⋮
10	55	66	132	253
⋮	⋮	⋮	⋮	⋮
20	210	231	462	903
⋮	⋮	⋮	⋮	⋮
n	$\frac{n(n+1)}{2}$	$\frac{(n+1)(n+2)}{2}$	$(n + 1)(n + 2)$	$2n^2 + 5n + 3$

B. 6×6

11×11

21×21

$(n + 1) \times (n + 1)$

The generalizations for the ■s and □s are $\frac{n(n+1)}{2} + \frac{(n+1)(n+2)}{2} = \frac{(n^2+n)+(n^2+3n+2)}{2} = \frac{2n^2+4n+2}{2} = n^2 + 2n + 1 = (n + 1)(n + 1)$. This equals the dimensions of the square formed by the ■s and □s.

C. 561, 1122, 2211

Have students explore by trial and error with a calculator to find $n = 32$ and then substitute for n in the generalizations. If $\frac{n(n+1)}{2} = 528$ ■s, then $n = 32$, $\frac{(n+1)(n+2)}{2} = 561$ □s, $(n + 1)(n + 2) = 1122$ ■s, and $2n^2 + 5n + 3 = 2211$ blocks.

D. The ■s can be combined (rotate one group 90°) into a rectangle with dimensions $(n + 2) \times (n + 1)$, which is the same as the generalization for the ■s. For the ■s, if you had a second group the same size, you could make a rectangle with dimensions $(n + 1) \times n$. Since $(n + 1) \times n$ would be twice as big as the original arrangement, you must divide by 2 to get the generalization for the ■s: $\frac{n(n+1)}{2}$.

E. Add each of the three generalizations:

Starting with $\frac{n(n+1)}{2} + \frac{(n+1)(n+2)}{2} + (n + 1)(n + 2)$,

(1) Get a common denominator: $\frac{n(n+1)}{2} + \frac{(n+1)(n+2)}{2} + \frac{2(n+1)(n+2)}{2}$;

(2) Expand terms in the numerator by multiplying: $\frac{(n^2+n)+(n^2+3n+2)+2(n^2+3n+2)}{2} = \frac{n^2+n+n^2+3n+2+2n^2+6n+4}{2}$;

(3) Combine like terms: $\frac{4n^2+10n+6}{2}$; and

(4) Divide each numerator term by 2: $2n^2 + 5n + 3$.

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